Mathematics Instructional Cycle Guide

F-BF.3- Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx) and f(x + k) for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effect on the graph using technology.

Created by Briana Visone, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Build new functions from existing functions

F-BF.3- Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx) and f(x + k) for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effect on the graph using technology. Include recognizing even and odd functions from their graph and algebraic expressions for them.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP6 Attend to Precision
MP7 Look for and make use of structure

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(Pages 2 – 5)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(Pages 6 – 14)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(Pages 15 – 24)**
- Supporting lesson materials **(Pages 25 – 29)**
- Precursory research and review of standard **F-BF.3** and assessment items that illustrate the standard **(Pages 30 – 32)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **Transformations Checkpoint** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- **Graph paper, Handouts, Calculators(optional), Chart paper with grids**

TIME NEEDED

Transformations Checkpoint administration: **15 minutes**
Follow-Up Lesson Plan: **2 45 minute lessons**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

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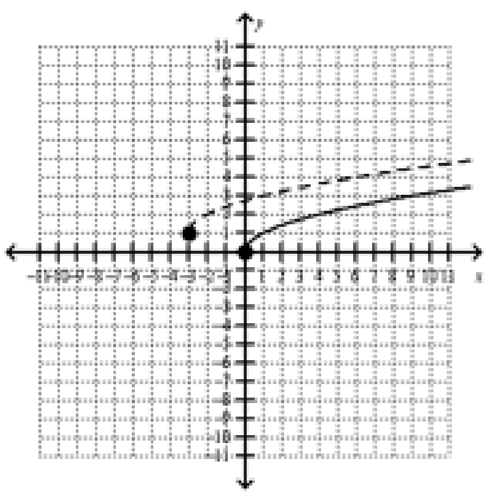
<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">Transformations Checkpoint:

Gina, Rowena, and Jordan were each asked to graph a function by transforming its parent function. The parent function is the solid graph and the transformation is the dashed graph. Decide if you agree with their transformations. Be sure to justify your answer.

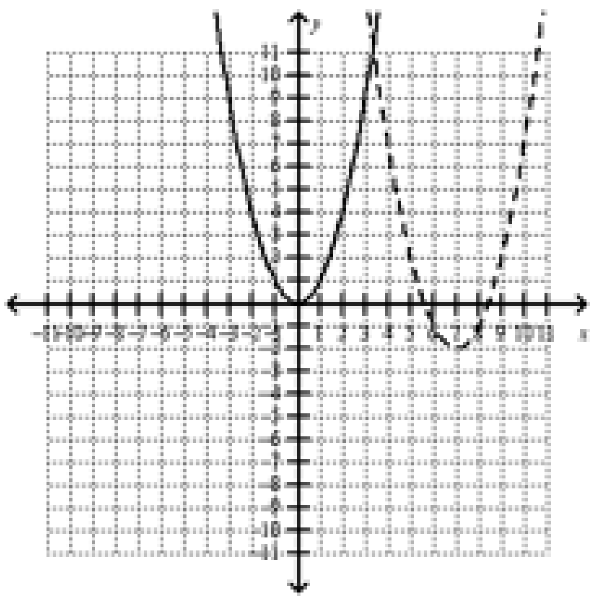
(a) Gina was given the function f(x) = $\sqrt{x-3}+1$
</td><td>CT Core Standard:</td><td>F-BF.3- Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx) and f(x + k) for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effect on the graph using technology.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>Do students understand how to transform points on a parent function?
Can students recognize errors in other student's transformations?
Are students able to explain in writing why a transformation is correct or incorrect?</td></tr>
</table>

Do you agree or disagree with Gina? (circle one)

Agree Disagree

Justify your answer in full sentences.

(b) Rowena was given the function g(x) = $-(x-7)^2+2$

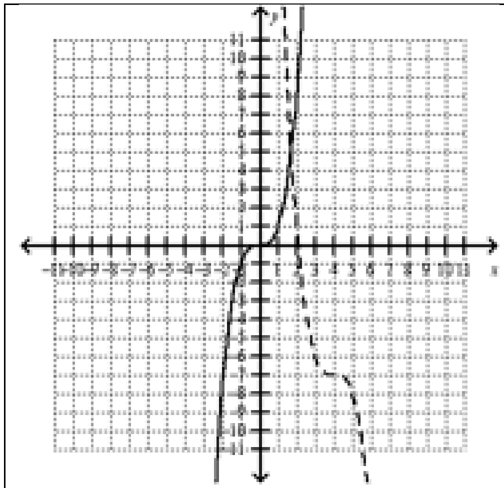


Do you agree or disagree with Rowena? (Circle one)

Agree Disagree

Justify your answer in full sentences.

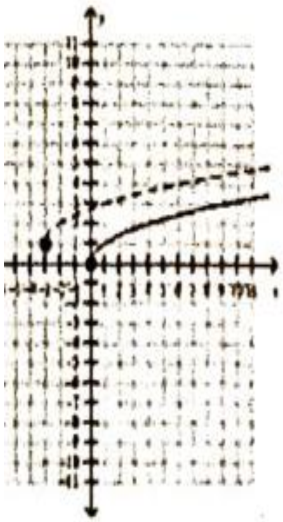
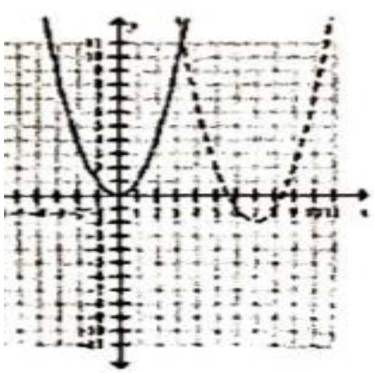
(c) Jordan was given the function $f(x) = -(x - 4)^3 - 7$



Do you agree or disagree with Jordan? (Circle one)

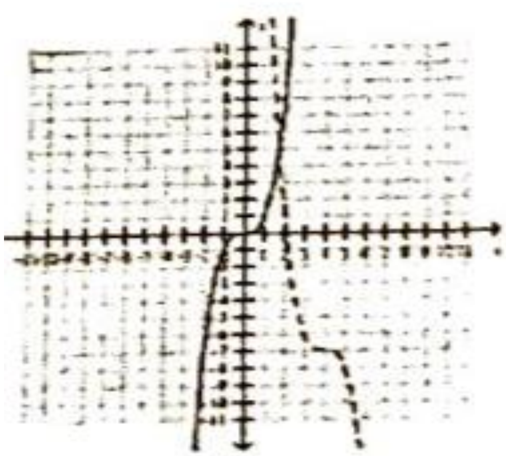
Agree Disagree

Justify your answer in full sentences.

Getting Started	
Student Response Example	**Indicators**
was given the function $f(x) = \sqrt{x-3}+1$ (3, 1)  Do you agree or disagree with Paul? (Circle one) Agree Disagree Justify your answer in full sentences. Right 3, up 1 ine was given the function $g(x) = -(x-7)^2+2$ (7, 2)  Do you agree or disagree with Yasmine? (Circle one) Agree Disagree Justify your answer in full sentences. Right 7, up 2	• The student may know the process but cannot explain why a solution is correct or incorrect. The understanding is only based on memorizing a shift of a graph. There is no algebraic understanding as to why a shift is done in a particular way. • The student is able to find the new origin point, but cannot explain why the shift is occurring. • This student has a very basic understanding of transformations of functions. The student knows the graph is supposed to be shifted, however he/she does not have the conceptual understanding of why the graph is shifted in a particular way. • This student is only able to state what the shift is and not the error being made. The getting started student also may not pick up on all mistakes made by the graph such as the reflection over the x-axis.

(c) Alexis was given the function $f(x) = -(x-4)^3 - 7$

(4 , -7)



Do you agree or disagree with Alexis? (Circle one)

Agree Disagree

Justify your answer in full sentences.

Left 4 , down 7

In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
If a student does not switch the sign of the x-value when finding the new origin, ask the student to plug it into the function. For example, if given $f(x) = (x-2)^2 + 5$ and the student states the origin point to be (-2, 5), ask the student to plug -2 into the equation. Do you get the y-value of 5? Then have the student plug 2 into the equation. This will show the students that f(-h) equals zero when we take the opposite sign and thus find the new origin point. Similarly, if the student is given the function $f(x) = -(x-2)^2 + 5$ and did not flip the graph upside down, have the student evaluate f(1) and f(3) to see what is happening to the y-values at those points.	Watch the below video of shifting a square root function by clicking the link below. http://ctdreamteam.learnzillion.com/lessons/3190-graph-square-root-functions-using-transformations

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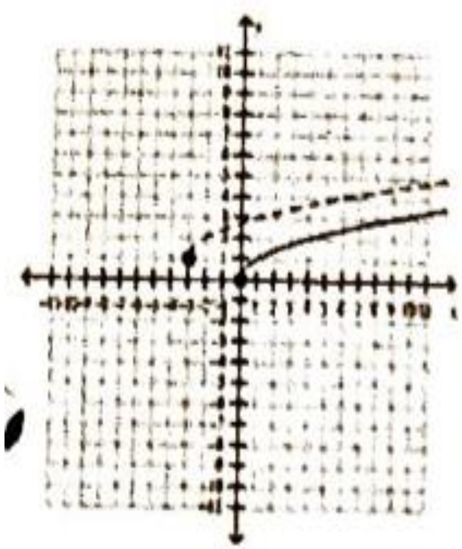
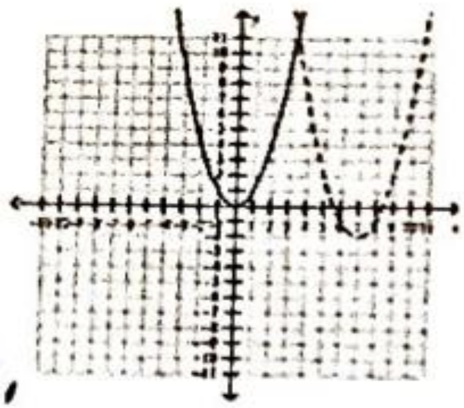
Questions: Why did you transform the function in the way you did? How did you find the origin/starting point? Why do we change the sign of the x-value when finding the origin/starting point? Why does the y-value of the origin/starting point stay the same?	

Developing	
Student Response Example	**Indicators**
 (1) Paul was given the function $f(x) = \sqrt{x-3}+1$ 3, 1 Do you agree or disagree with Paul? (Circle one) Agree Disagree Justify your answer in full sentences. He didnt chang the sign.	• If a student is able to recognize the error in the first example they have a basic understanding of the transformations being done. In the second example, the student knew the graph was incorrect, but was not specific about the error. She stated, "He changed the wrong sign." Based on the work provided, the student knew that the origin point should be (7, 2), however she did not mention the graph being a reflection over the x-axis. She also was not able to extend her knowledge to question 3. She could not compare the graph with the transformation. • The student can successfully explain a basic transformation, however whenever a transformation involving multiple steps is given, the student cannot adequately explain multiple errors. • The student may understand the process of the transformation, however cannot adequately explain why a transformation is incorrect. • This student is able to do a single transformation such as up/down, left/right, reflection, however if more than one transformation is required, the student confuses the different transformations and makes errors.

(2) Yasmine was given the function $g(x) = -(x-7)^2 + 2$ 7, 2



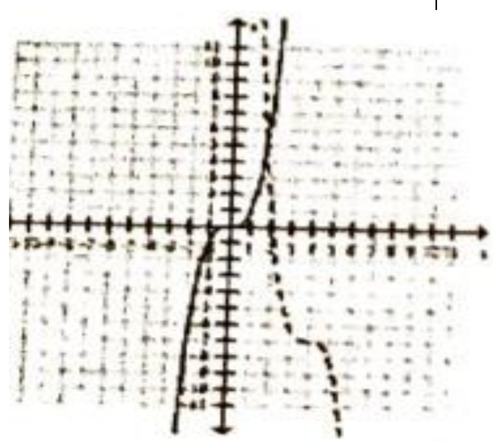
Do you agree or disagree with Yasmine? (Circle one)

Agree Disagree

Justify your answer in full sentences.

He changed the wrong sign.

c) Alexis was given the function $f(x) = -(x-4)^3 - 7$



Do you agree or disagree with Alexis? (Circle one)

Agree Disagree

Justify your answer in full sentences.

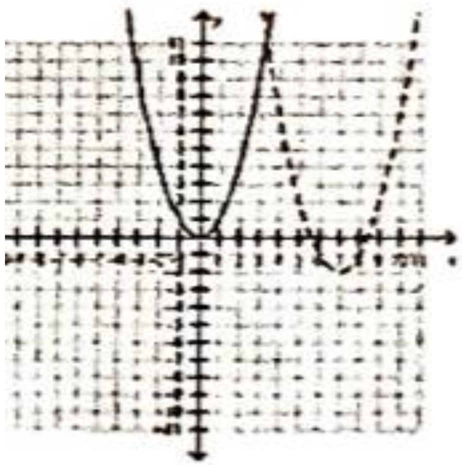
She plotted in the wrong place.

In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
You could give the student a similar problem and have him/her list its attributes(types of transformations that are being asked, up/down, left/right, upside down) before even looking at the graph. Questions: • What will the origin be? • Are there any other types of transformations on this graph? After eliciting this information, then have the student draw a sketch of what the graph should look like before analyzing someone else's work.	Watch the below video of shifting a square root function by clicking the link below http://ctdreamteam.learnzillion.com/lessons/3190-graph-square-root-functions-using-transformations

<table>
<tr><th colspan="2">Got it</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td>

Yasmine was given the function g(x) = $-(x-7)^2+2$

(7,2)



Do you agree or disagree with Yasmine? (Circle one)

Agree Disagree

Justify your answer in full sentences.

I disagree with Yasmine because she plotted the graph incorrectly, the ~~graph~~ vertex ~~was~~ was suppose to be at (7,2) and instead she plotted at (7,-2). Also the function was a negative so the parabola should be going down instead of going up.

</td><td>

- In all three examples, the student correctly agreed or disagreed with the three student examples and gave a correct reason for the mistakes made. The student was able to recognize when the transformation was done incorrectly.
- The student can explain the mistakes of the wrong answers and explain the correct answer. The student can look at a function they have never graphed and recognize whether it is transformed correctly based on the original graph.
- The student has a strong understanding of shifting transformations. He can state the origin point and recognize when multiple errors are made.
- While the student can accurately describe the errors, whether he knows why the process for the shift is done is not necessarily clear from his responses.

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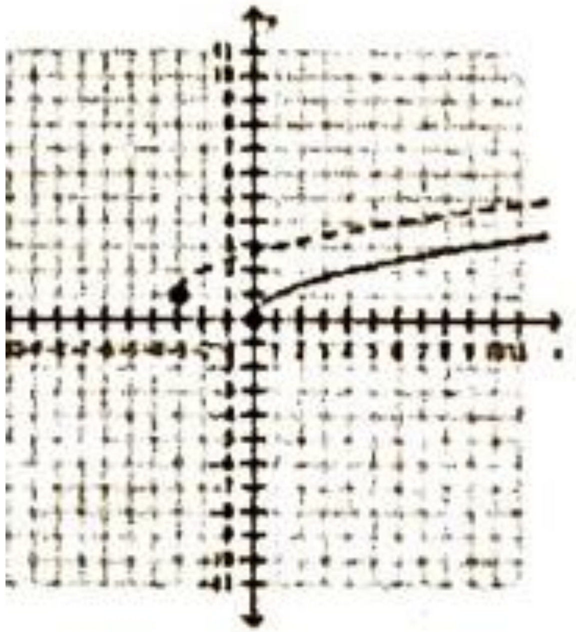
Paul was given the function $f(x) = \sqrt{x-3}+1$

(3,1)



Do you agree or disagree with Paul? (Circle one)

Agree

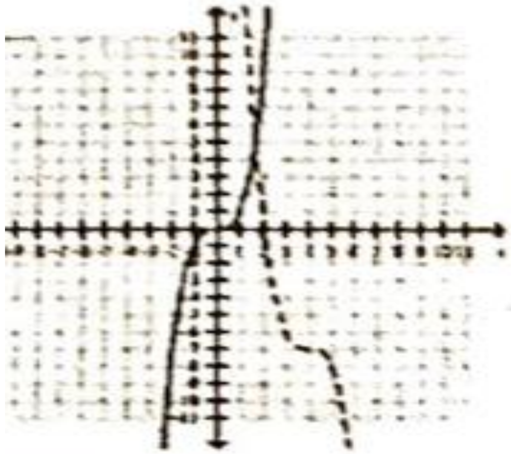
Disagree

Justify your answer in full sentences.

I disagree with Paul because Paul plotted at (-2,1) when he was suppose to plot at (3,1), so the graph was incorrect

Alexis was given the function $f(x) = -(x-4)^3 - 7$

(11, -7)



Do you agree or disagree with Alexis? (Circle one)

Agree Disagree

Justify your answer in full sentences.

I agree with Alexis because she correctly plotted the transformation, which was suppose to be at (11, -7), and that's where it was plotted.

In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
Have students conjecture what would happen if given f(kx). Have the students create tables to figure out what type of transformation this would be. Give students a piecewise function and ask the student to transform the function. If they can transform a function they have never graphed before, they understand the process of transformations quite well.	Have students watch the video below on graphing cube roots functions and then have them apply the rules of transformations. **http://ctdreamteam.learnzillion.com/lessons/3115-graph-cube-root-functions**

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Students will be able to graph any function using transformations
Content Standard(s):	F-BF.3- Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx) and f(x + k) for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effect on the graph using technology.
Targeted Practice Standard :	*MP6 Attend to Precision* *MP7 Look for and make use of structure*

Mathematical Goals	Success Criteria
1) Understand that the graphing of different functions is connected. *2) Understand that no matter the function, the process that is used to transform the graph of a function is the same.*	*Students will be able to graph any function of the form f(x) = f(x – h) + k where h and k are integers, given the original function.*

Launch (Probe and Build Background Knowledge)

Purpose:

Have the students graph the following functions:

1. **$f(x) = x^2$**
2. **$g(x) = |x|$**
3. **$h(x) = x^3$**
4. **$f(x) = \sqrt{x}$**

Elicit from the students how they did this. If they used a table, ask the students how they chose which x-values to plug into the function. (A graphing calculator may or may not be used.)

Instructional Task

Engage (Setting Up the Task):

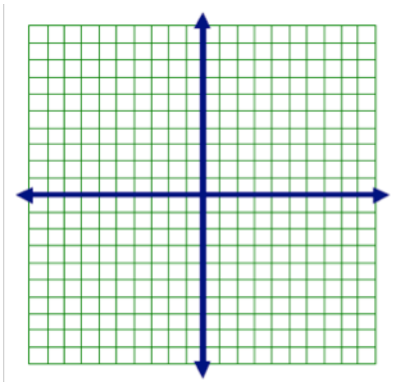
Activity: Jigsaw

Break the students into groups of four students. Each group will be given a different set of functions to graph. The students will use a table to find points on the graph and then graph the transformation.

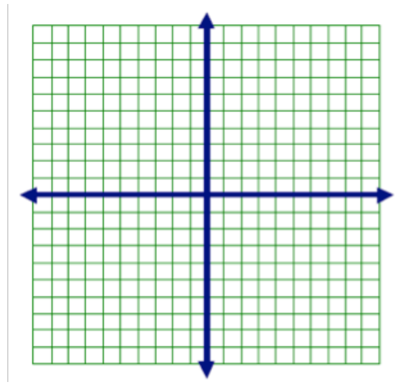
Group 1:

On the axes provided, graph the following functions. You may use the tables to help you.

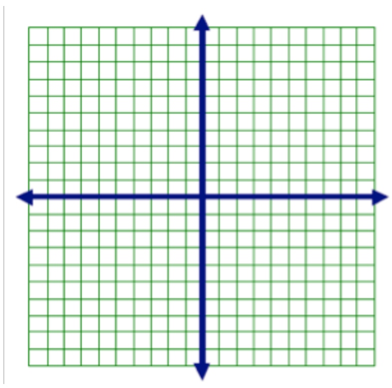
1. $f(x) = x^2 + 3$



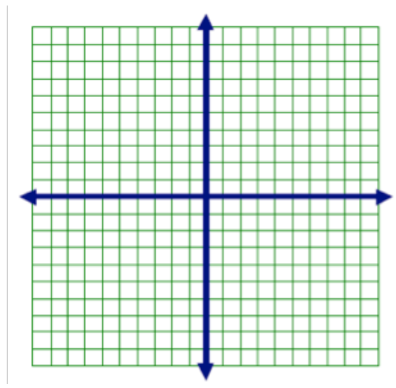
2. $f(x) = x^2 - 5$



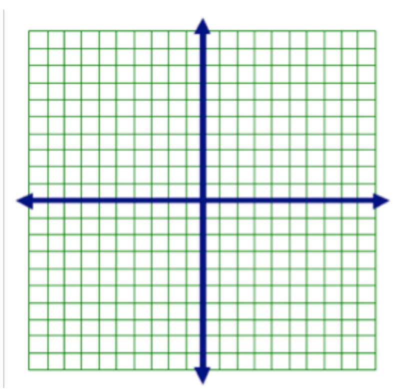
3. $g(x) = |x| + 1$



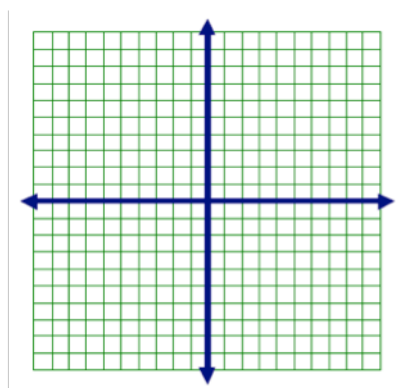
4. $g(x) = |x| - 7$



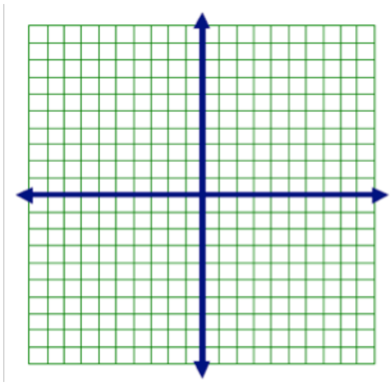
5. $h(x) = x^3 + 2$



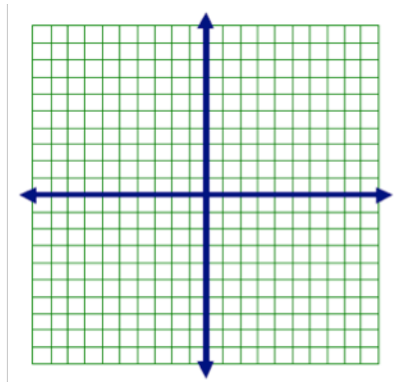
6. $h(x) = x^3 - 3$



7. $g(x) = \sqrt{x} + 6$



8. $g(x) = \sqrt{x} - 4$



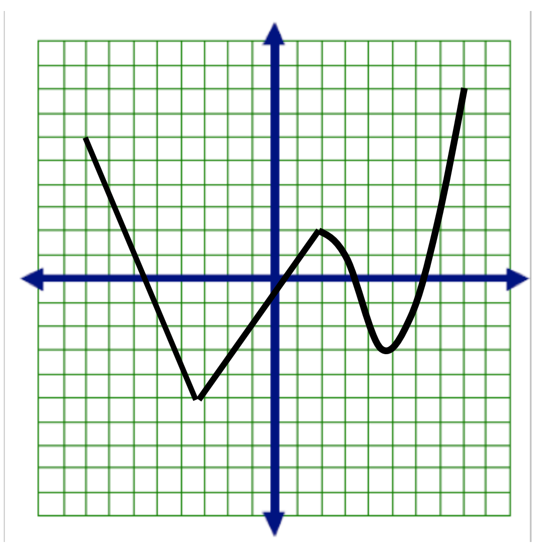
What pattern do you notice?

Did you need to use the table for every function?

Where did your starting point/origin end up?

Why do you think that happened?

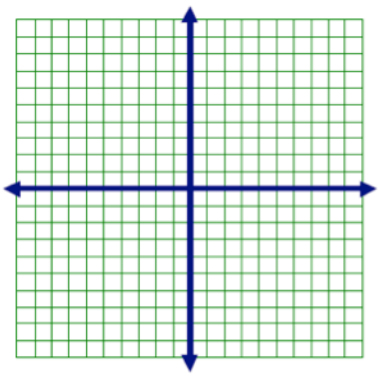
Provide the following piecewise function. Ask the students to graph y = f(x) – 3.



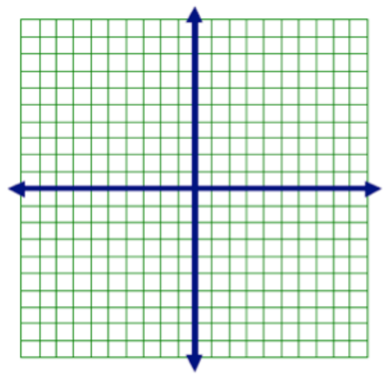
Group 2:

On the axes provided, graph the following functions. You may use the tables to help you.

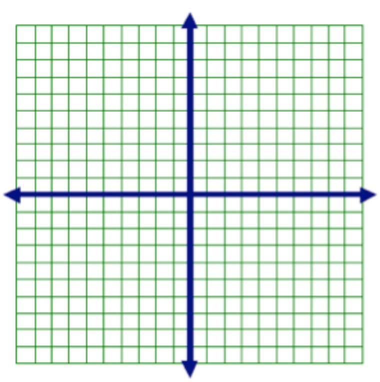
1. $f(x) = (x + 3)^2$



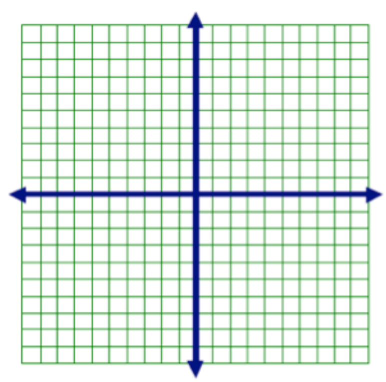
2. $f(x) = (x - 4)^2$



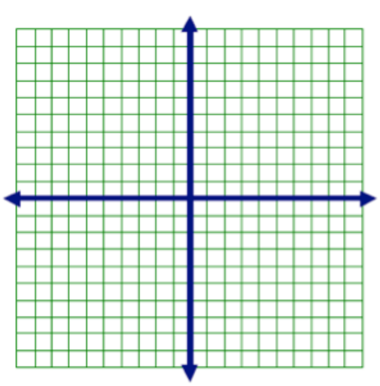
3. $g(x) = |x + 8|$



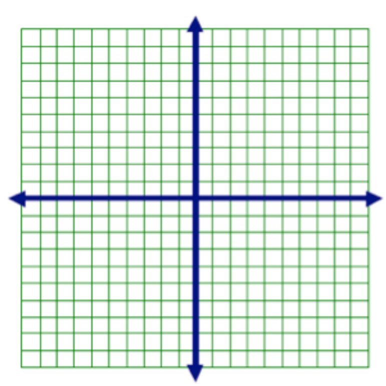
4. $g(x) = |x - 5|$



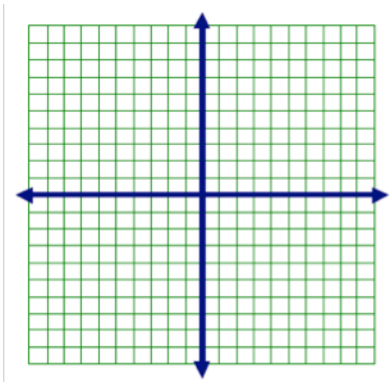
5. $h(x) = (x + 1)^3$



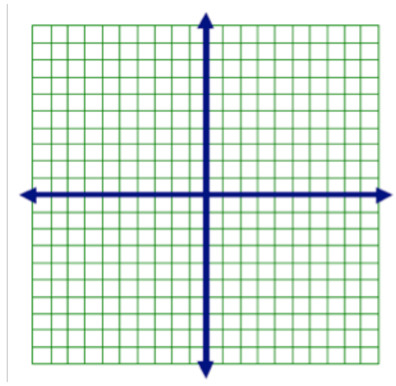
6. $h(x) = (x - 7)^3$



7. $g(x) = \sqrt{x+6}$



8. $g(x) = \sqrt{x-2}$



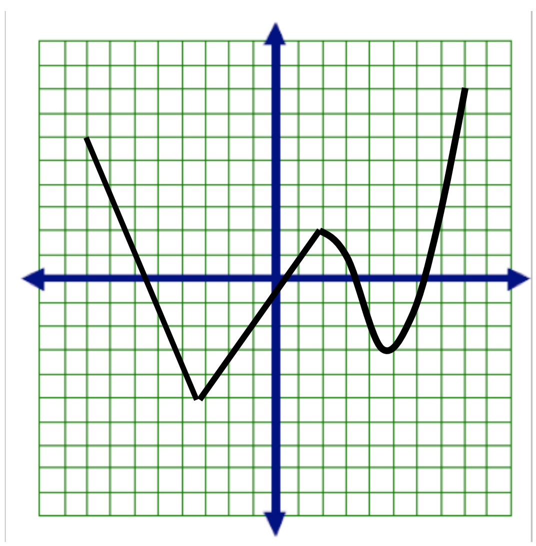
What pattern do you notice?

Did you need to use the table for every function?

Where did your starting point/origin end up?

Why do you think that happened?

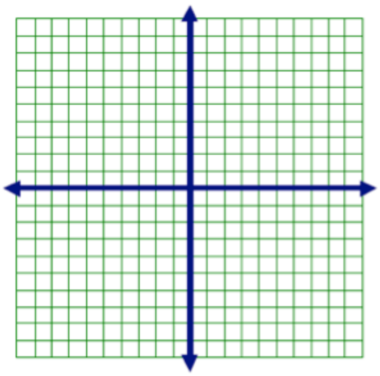
$y = f(x)$ is graphed below. Graph $y = f(x - 3)$ on the same set of axes.



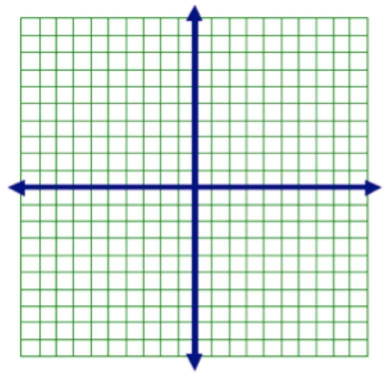
Group 3:

On the axes provided, graph the following functions. You may use the tables to help you.

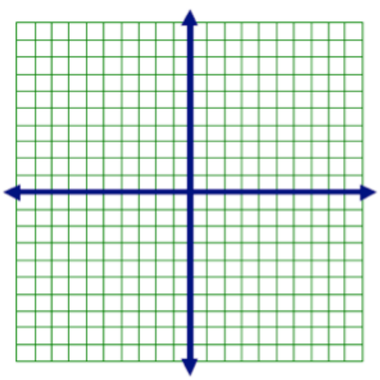
1. $f(x) = -x^3$



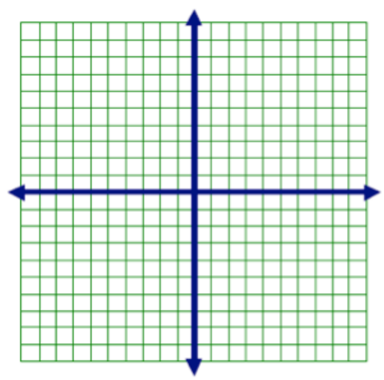
2. $f(x) = -x^2$



3. $g(x) = -|x|$



4. $g(x) = -\sqrt{x}$



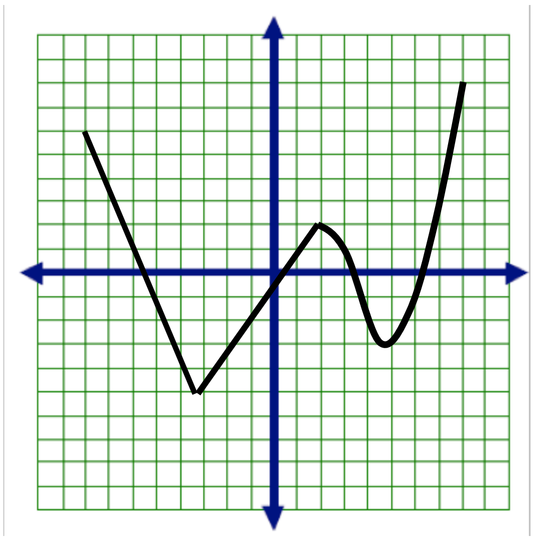
What pattern do you notice?

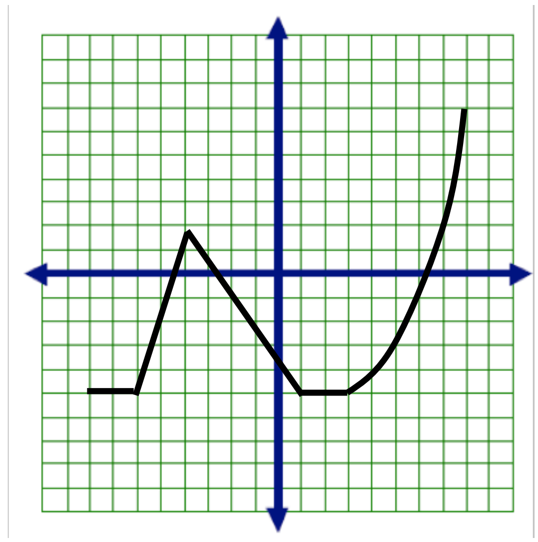
Did you need to use the table for every function?

Where did your starting point/origin end up?

Why do you think that happened?

In the following problems, y = f(x) is graphed. On the same set of axes, graph y = -f(x).

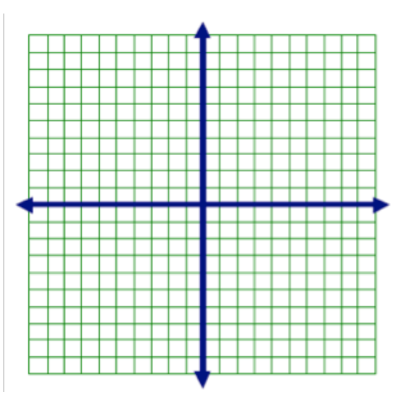




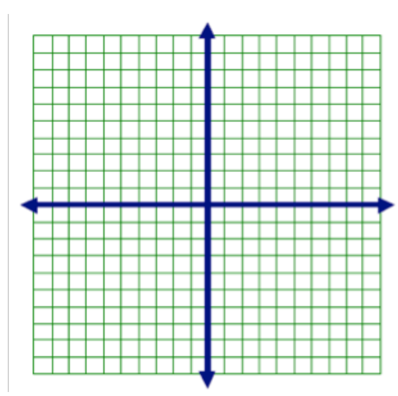
Group 4:

On the axes provided, graph the following functions. You may use the tables to help you.

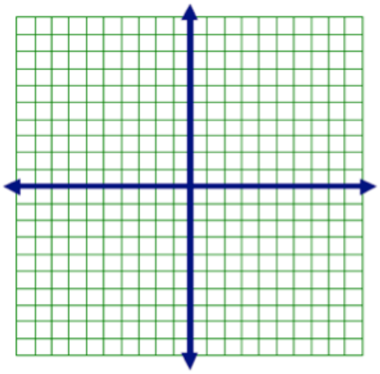
1. $f(x) = (x + 5)^2 + 1$



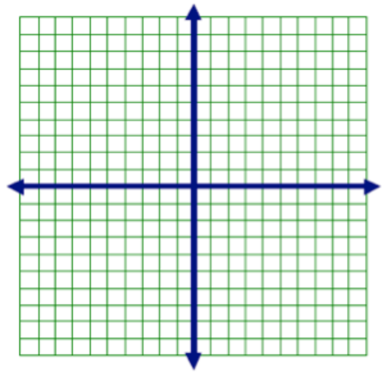
2. $f(x) = (x - 2)^2 + 3$



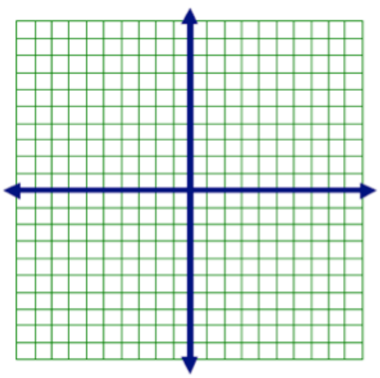
3. $g(x) = |x + 6| - 5$



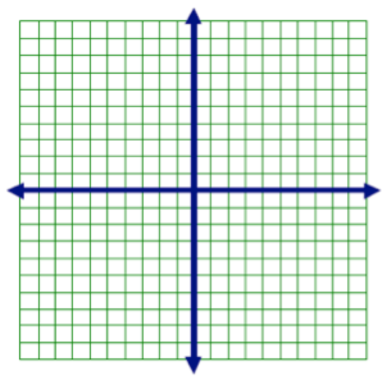
4. $g(x) = |x - 4| - 7$



5. $h(x) = \sqrt{x-3}+4$



6. $h(x) = \sqrt{x+1}-6$



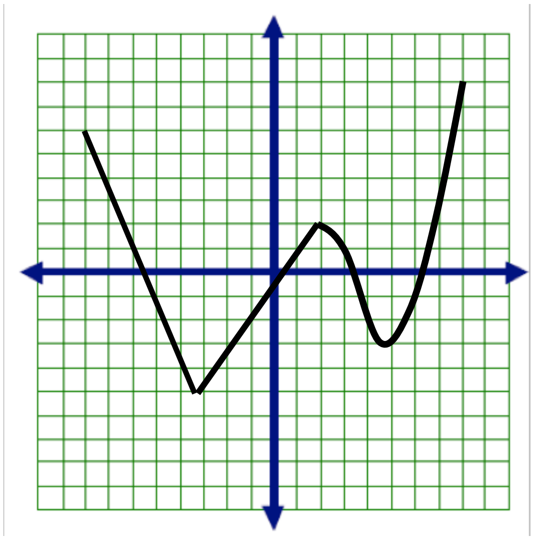
What pattern do you notice?

Did you need to use the table for every function?

Where did your starting point/origin end up?

Why do you think that happened?

Below is the graph y = f(x). On the same set of axes, graph y = f(x – 3) + 2.



Explore (Solving the Task)

Regroup the students. Put one student from each of the previous groups into a new group. These students are the "experts" on what they just did. They will take a few minutes to share their work with their group members.
As a group, have them answer the following questions:

What happens to a function when a number is added to the outside of it? For example, $f(x) = x^2 + 10$. What if the number is negative instead of positive? What kind of shift happens? Why do you think that happened?

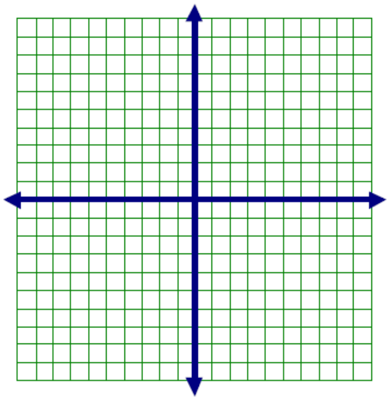
What happens to a function when a number is added to the inside of a function? For example, $f(x) = \sqrt{x+11}$.
What if the number is negative instead of positive? What kind of shift happens? Why do you think that happened?

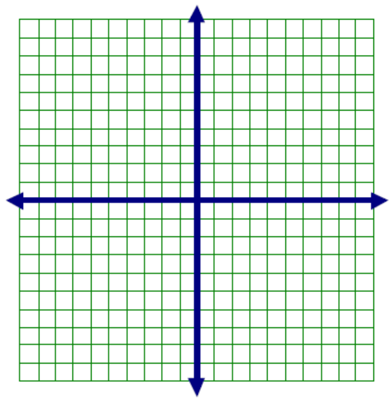
What happens when a function is negated? For example, $f(x) = -|x|$. What changes within the function? What stays the same?

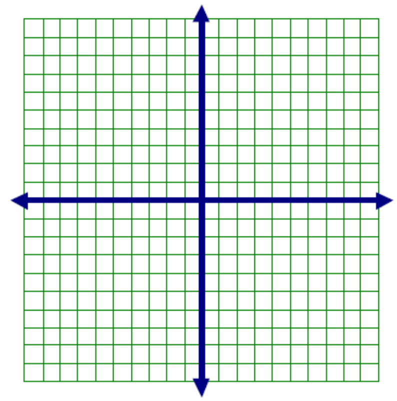
What happens when a number is added to both the inside and outside of a function? For example, $f(x) = (x + 8)^3 + 15$. What if the numbers are negative instead of positive? What kind of shift happens? Why do you think this happens?

In your groups, using the shifting rules you just discovered, graph the following functions without a calculator or table:

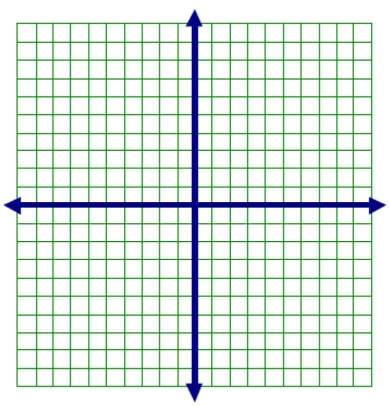
1. $f(x) = x^2 + 1$
2. $f(x) = -|x - 4|$
3. $G(x) = -\sqrt{x} + 3$

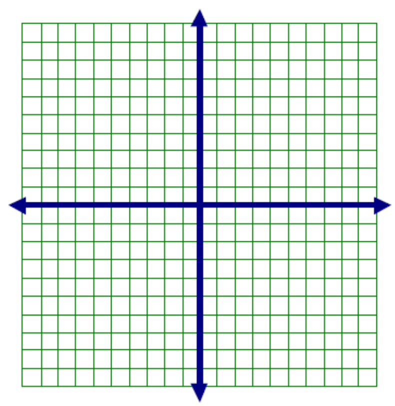






4. $f(x) = (x - 7)^3 + 2$
5. $h(x) = -(x + 2)^2 + 7$





Have each group plot one of the above graphs on a piece of large graph paper.

Elaborate (Discuss Task and Related Mathematical Concepts)
Group discussion:
Put each of the graphs up on the board and have students decide if they agree or disagree with each of the graphs.

Discuss the transformations. What do you do when a number is inside a function? Added or subtracted to a function? The function is negated?

Go through each of the five examples and decide if they are correct.
Be sure to reinforce that the transformation is exactly the same no matter which function you are graphing.
Discuss common misunderstandings with students.

Checking for Understanding

Purpose:

Give the following exit ticket:

What is the origin point or starting point for the following functions. Justify your answer.:

1. $f(x) = |x + 6|$
   Starting Point/Origin: Explain your answer:

2. $f(x) = x^2 + 7$
   Starting Point/Origin: Explain your answer:

3. $f(x) = (x + 4)^3 – 12$
   Starting Point/Origin: Explain your answer:

4. $f(x) = -\sqrt{x} + 5$
   Starting Point/Origin: Explain your answer:

5. $h(x) = -|x – 14| - 11$
   Starting Point/Origin: Explain your answer:

Common Misunderstanding

Purpose: Make students aware of possible mistakes and misunderstandings by showing them possible errors they may make.

Show examples of student work from the checkpoint to show the errors that can occur.

-Not changing the sign of the x-value of the origin point.

-When a negative is in front of the function, switching the sign of the origin point rather than flipping the graph upside down.

-Reflecting a graph over the y-axis instead of the x-axis when a function is negated.

Checking for Understanding

Questions:

-Why do we change the sign of the inside value (x-value) but not the outside (y-value)?

- Why does the inside number cause a horizontal shift?

-Why does the number being added or subtracted to the function cause a vertical shift?

Give another exit ticket or set of do now problems.

What is the starting point/origin point for these functions? Justify your answer.

1. f(x) = $\sqrt{x-2}+3$
   Starting point/origin: Explain your reasoning:

2. f(x) = $-x^2 + 5$
   Starting point/origin: Explain your reasoning:

3. f(x) = $|x| - 7$
   Starting point/origin: Explain your reasoning:

4. f(x) = $-(x + 3)^3$
   Starting point/origin: Explain your reasoning:

5. h(x) = $-(x + 10)^2 - 15$
   Starting point/origin: Explain your reasoning:

Closure

Purpose:

Give students a similar set of problems to the checkpoint. Student will analyze other students work and find mistakes. Then give the students an exemplar solution. Have students read though the solutions and explanations. The explanations are very helpful to students understanding their errors and understanding. Have students reflect on their solutions and how they could improve their work in the future by answering the following questions:

1. *I am a able to accurately graph $y = x^2$, $y = x^3$, $y = |x|$, $y = x^3$*
   Not at all Absolutely
   1 2 3 4 5
2. I am able to find the origin point/starting point of a graph.
   Not at all Absolutely
   1 2 3 4 5
3. I know what to do when there is a negative in front of a function.
   Not at all Absolutely
   1 2 3 4 5
4. I am able to find mistakes in other student's work.
   Not at all Absolutely
   1 2 3 4 5

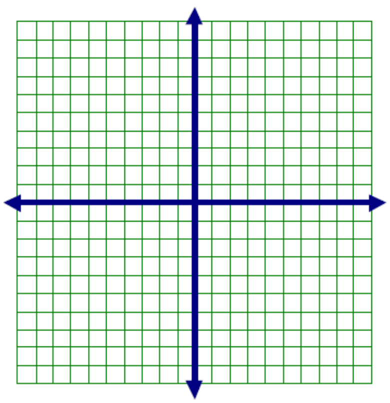
5. I am able to explain why a graph is translated correctly or not.
   Not at all Absolutely
   1 2 3 4 5
6. I know why I am supposed to change the sign of the x-value but not the y-value when translating.
   Not at all Absolutely
   1 2 3 4 5

Extension Task

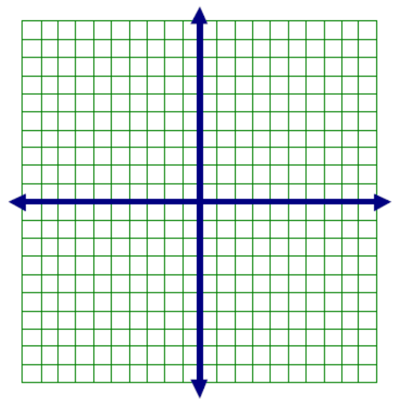
Similar to the jigsaw activity, have students walk through the following problems using a table to find a general rule when a number is multiplying a function on the inside.

On the axes provided, graph the following functions. You may use the tables to help you.

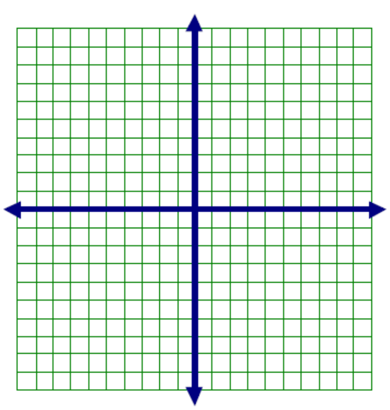
1. $f(x) = (2x)^2$



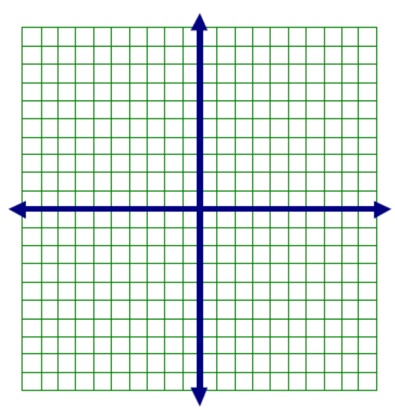
2. $f(x) = \left(\frac{1}{2}x\right)^2$



3. $g(x) = |3x|$

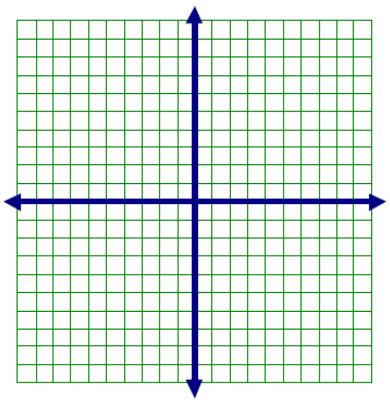


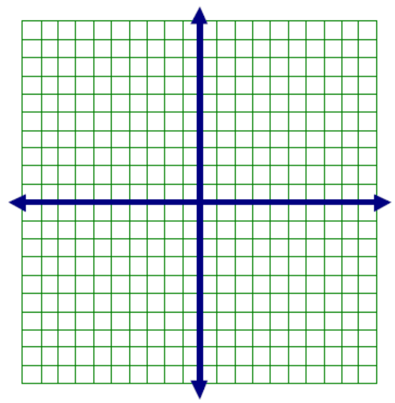
4. $G(x) = \left|\frac{1}{3}x\right|$



5. $h(x) = (2x)^3$

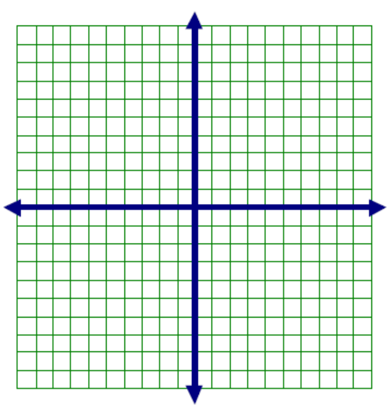
6. $h(x) = \left(\frac{1}{4}x\right)^3$

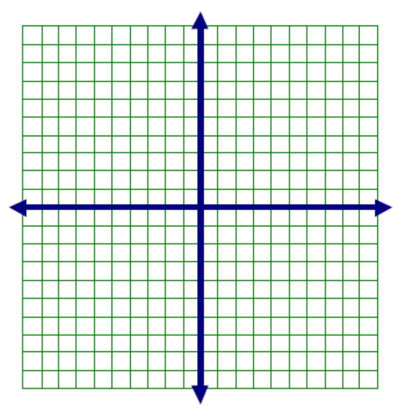




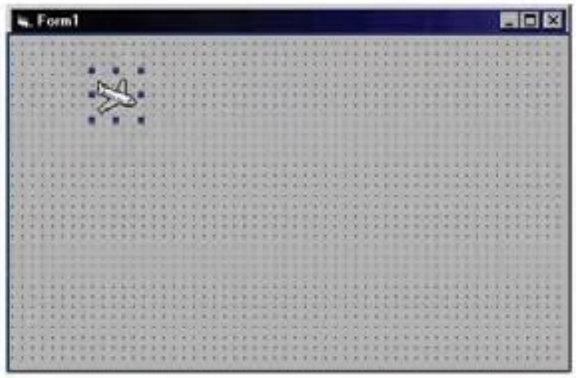
7. $g(x) = \sqrt{5x}$

8. $g(x) = \frac{1}{2}\sqrt{x}$





Application Extension Task:



APPLICATION

Kari's assignment in her computer programming course is to simulate the motion of an airplane by repeatedly translating it across the screen. The coordinate system in the software program is shown at right. In this program, coordinates to the right and down are positive. The starting position of the airplane is (1000, 500), and Kari would like the airplane to end at (7000, 4000). She thinks that moving the airplane in 15 equal steps will model the motion well.

a. What should be the airplane's first position after (1000, 500)?

b. If the airplane's position at any time is given by (x, y), what is the next position in terms of x and y?

c. If the plane moves down 175 units and right 300 units in each step, how many steps will it take to reach the final position of (7000, 4000)?

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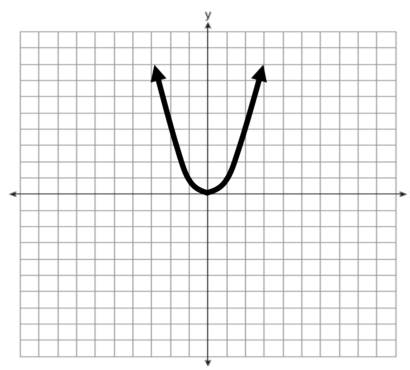
<table>
<tr><th colspan="2">Research and review of standard</th></tr>
<tr><th>Content Standard(s):</th><th>Standard(s) for Mathematical Practice:</th></tr>
<tr><td>Build new functions from existing functions

F-BF.3- Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx) and f(x + k) for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effect on the graph using technology. Include recognizing even and odd functions from their graph and algebraic expressions for them.</td><td>MP6 Attend to Precision

MP7 Look for and make use of structure</td></tr>
<tr><th>Smarter Balanced Claim</th><th>Smarter Balanced Item</th></tr>
<tr><td>Claim # 1 – Concepts & Procedures

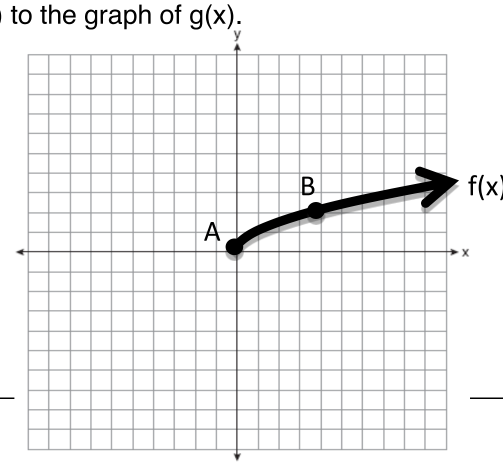
“Students can explain and apply mathematical concepts and interpret and carry out mathematical procedure with precision and fluency.”</td><td>Practice test Item # 1
The graph of y = x^2 is shown on the grid.

Drag the graph to show y = $(x – 4)^2 + 2$.



Practice test item # 16

The graph of g(x) = $\sqrt{x-2}+3$ is a translation of the graph of f(x).

Plot the translations of points A and B from the graph of f(x) to the graph of g(x).

</td></tr>
</table>

CPR Pre-Requisites	**Conceptual Understanding and Knowledge** • All graphs of the form y = f(x – h) + k is based on the graph y = f(x), translated horizontally h units and vertically k units where h and k are rational numbers. • No matter function, quadratic, square root, line, absolute value, cubic, etc., the new origin is the same translation. **Procedural Skills** • Students must have the ability to graph the parent functions on the coordinate plane by creating a table of values and scaling it accurately. • Students must be able to plot points accurately on the coordinate plane. **Representational** • Students must be able to understand and recognize the equation of a function and its graph on the coordinate plane. **Conventions and Social knowledge** • Function Notation (f(x), g(x), etc) • Square root notation • Parabola • Function • Radical • Cubic

Standards Progression **Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*		
Pre-Requisite Standards	**Co-Requisite Standards**	**Future Standards**
F-IF.7a – Graph linear and quadratic functions and show intercepts, maxima and minima.	*F-IF.7b – Graph square root, cube root, and piecewise functions, including step functions and absolute value functions.*	*F-IF.7e – Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude*

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- When given a function of the form y = f(x – h) + k, where h and k are positive numbers, students tend to translate the graph left h units and up k units (or use the vertex point (-h, k)). They forget to change the sign of the x-value.
- When given a function of the form y = k + f(x – h), where h and k are positive numbers, students will translate the graph right k units and down h units.
- When given a function of the form y = f(x – h) + k, where h and k are positive numbers, students will translate the graph right k units and up h units (or use the vertex point (k, h)).
- When given a function of the form y = -f(x – h) + k, students will not flip the graph upside down, but just graph the function as if the negative sign were not there.
- When given a function of the form y = -f(x – h) + k, where h and k are positive, students will translate the graph left h units and up k units.
- When given a function of the form y = -f(x – h) + k, where h and k are positive, students will translate the graph right h and down k.
- When given multiple functions to graph, students confuse the equations of a function with other equations of a function. (For example, when given the equation of an absolute value function, the students will graph a parabola.)

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students struggle to distinguish between the equation of a parabola, square root, cubic, absolute value and line.
- Students may not have the skills to graph the parent functions.
- Students may struggle to plot points on a coordinate plane.
- Students may only have a basic understanding of what the function looks like graphically, but are unsure how to actually graph it accurately on the coordinate plane.

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- If the first graph a student is given to translate is a parabola, they will assume every graph they are given is a parabola.
- Since the formula f(x – h) + k translates the graph right and up for positive values of h and k, students will assume every translation will be right and up, no matter the sign of h and k.
- In Geometry, negating the x or y value will reflect a graph over the x or y axis, so rather than flip the graph upside down when given y = - f(x – h) + k, the students will reflect the graph over the x-axis or y-axis.