

Mathematics Instructional Cycle Guide

Fractions on the number line 3NF2a

Created by Kelly Palaia, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Developing an Understanding of Fractions as Numbers

NF 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP7 Look for and make use of structure.

MP6 Attend to precision

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings (**page 3**)
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (**pages 4-7**)
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (**pages 8-13**)
- Supporting lesson materials (**pages 14-20**)
- Precursory research and review of standard **3NF2a** and assessment items that illustrate the standard (**pages 21-23**)

HOW TO USE THIS DOCUMENT

- 1) Before the lesson, administer the **Plot 4/4 on the number line** [Mathematical Checkpoint](#) individually to students to elicit evidence of student understanding.
- 2) Analyze and interpret the student work using the [Student Response Guide](#)
- 3) Use the next steps or **follow-up lesson plan** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
- 4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- **Smart board or projector**
- **Appendix materials**
- **Dice**
- **Crayons**

TIME NEEDED

Plot 4/4 on the number line administration: **8 minutes**

Follow-Up Lesson Plan: **45 minutes**

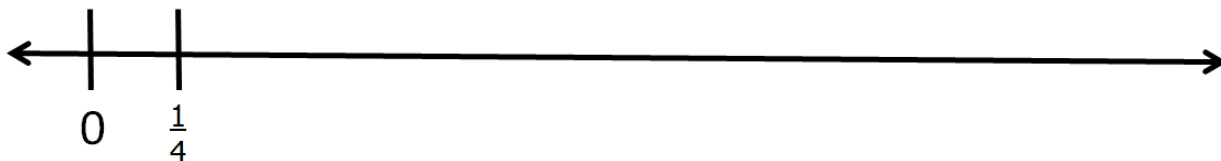
Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

Step 1: Elicit evidence of student understanding

Mathematical Checkpoint

Question(s)

Plot $\frac{4}{4}$ on the number line.



Explain to a friend how you knew where to put $\frac{4}{4}$.

Purpose

Developing an Understanding of Fractions as Numbers

3NF 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

- a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

CT Core Standard:

Target question addressed by this checkpoint:

Are students able to demonstrate an understanding that the number line needs to be partitioned into equal parts as determined by the denominator?

Step 2: Analyze and Interpret Student Work
Student Response Guide

Got It

Plot $\frac{4}{4}$ on the number line.

Explain to a friend how you knew where to put $\frac{4}{4}$.

first i put $\frac{4}{4}$ over there because i knew that i had to find what the total space was for the whole line and i know that all that space couldn't be filled up until $\frac{4}{4}$ so i measured the space in between the numbers and that's why i put $\frac{4}{4}$ there.

Developing

Plot $\frac{4}{4}$ on the number line.

Explain to a friend how you knew where to put $\frac{4}{4}$.

I knew where to put $\frac{4}{4}$ because $\frac{4}{4}$ is equal to 1 whole.

Getting Started

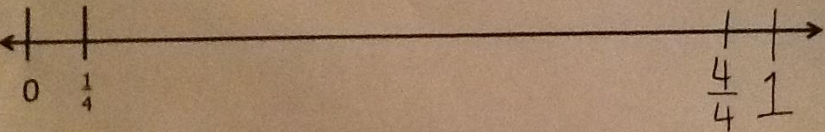
Mathematical Checkpoint

Plot $\frac{4}{4}$ on the number line.

Explain to a friend how you knew where to put $\frac{4}{4}$.

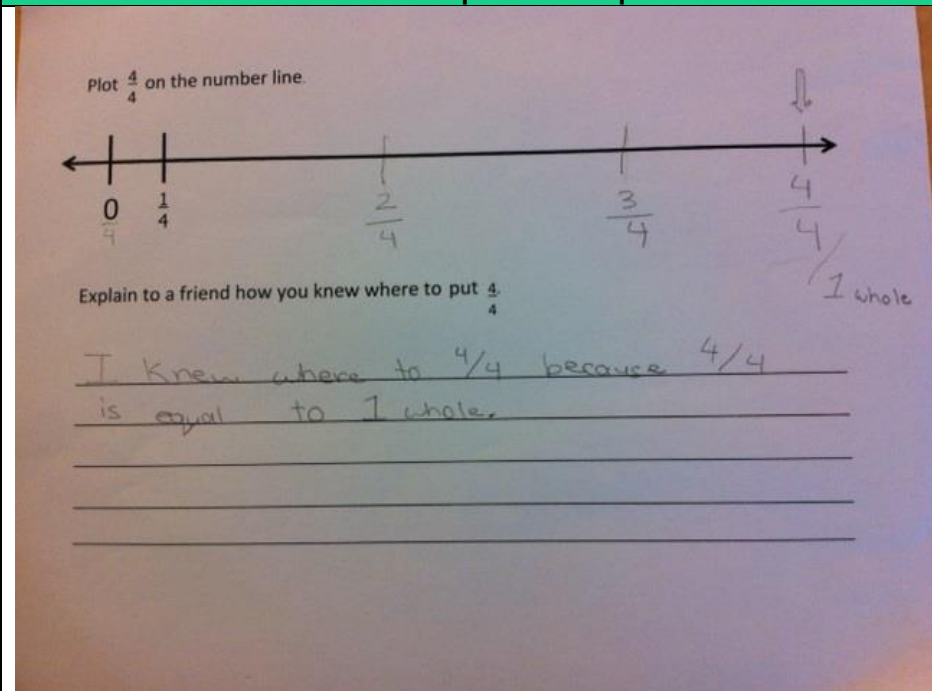
I knew where to put $\frac{4}{4}$ because I rememberd to put $\frac{4}{4}$ near the 1 because $\frac{4}{4} = 1$.

Getting Started

Student Response Example	Indicators
<p>Mathematical Checkpoint</p> <p>Plot $\frac{4}{4}$ on the number line.</p>  <p>Explain to a friend how you knew where to put $\frac{4}{4}$.</p> <p>I knew where to put $\frac{4}{4}$ because I remembered to put $\frac{4}{4}$ near the 1 because $\frac{4}{4} = 1$.</p>	<p><i>Does not understand that the denominator represents the number of equal parts that the line needs to be partitioned into</i></p> <p><i>Does not label the partitions accurately</i></p> <p><i>May change the denominator (adding one each time)</i></p> <p><i>Does not understand the order of fractions. (1/4, 2/4, 3/4...)</i></p> <p><i>When the numerator and denominator are the same they are equal to a whole or believes one whole must fall at the end of the number line.</i></p>
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
<p>How many equal parts does this number line need to be divided into? How do you know?</p> <p>How are $\frac{4}{4}$ and 1 related?</p> <p>What numbers are already on the number line? How can that help you solve the problem?</p>	<p>Identify a fraction as a point on a number line using area models https://learnzillion.com/lessons/1729-identify-a-fraction-as-a-point-on-a-number-line-using-area-models</p> <p>Give students rectangular shaped paper, have them practice folding into equal halves, fourths, eighths, thirds and sixths. Label each unit part.</p>

Developing

Student Response Example



Plot $\frac{4}{4}$ on the number line.

Explain to a friend how you knew where to put $\frac{4}{4}$.

I knew where to $\frac{4}{4}$ because $\frac{4}{4}$ is equal to 1 whole.

Indicators

Place fraction labels on the number line in correct order but spacing is not equal (for example; The space between $0/4$ and $\frac{1}{4}$ is equal to one fourth and must be equal to the other one fourth spaces on the number line)

In words students explain that you must partition the number line into equal parts but they did not partition accurately on the number line

Counting fractions in order

One whole always falls at the end of a number line

In the Moment Questions/Prompts

What do you notice about the space between 0 to $\frac{1}{4}$? How can you use that to help you partition the rest of the number line?

What do you notice about the spaces between your partitions?

Why did you place one whole at the end of the number line?

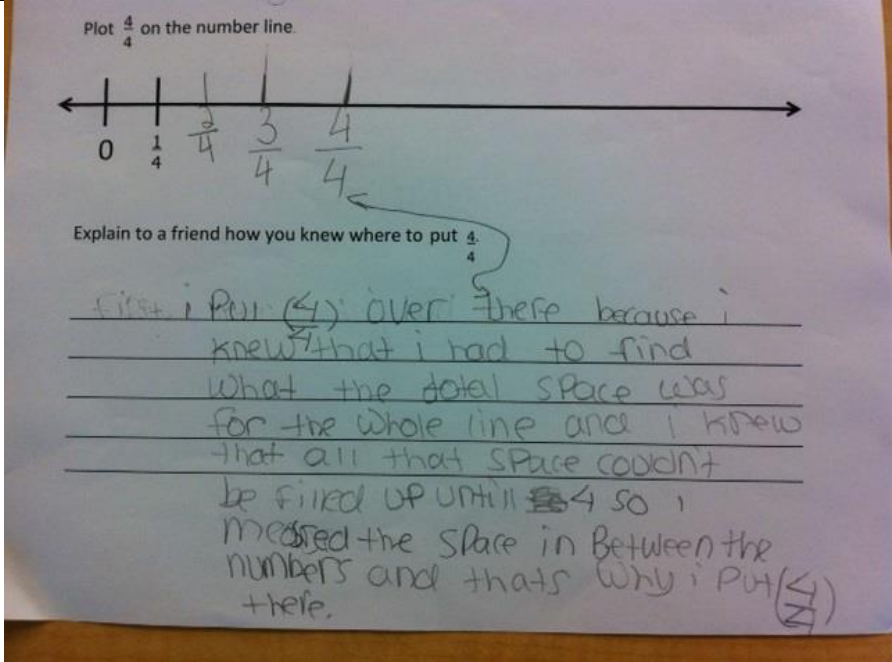
Closing the Loop (Interventions/Extensions)

[Identify a fraction as a point on a number line by dividing the number line into equal parts](https://learnzillion.com/lessons/1728-identify-a-fraction-as-a-point-on-a-number-line-by-dividing-the-number-line-into-equal-parts)

<https://learnzillion.com/lessons/1728-identify-a-fraction-as-a-point-on-a-number-line-by-dividing-the-number-line-into-equal-parts>

Folding paper into equal parts to learn how we split in $\frac{1}{2}$ first then fourths.

Draw a rectangle above the number line to represent one whole, students partition the area of the whole into fourths and make a connection to the number line.

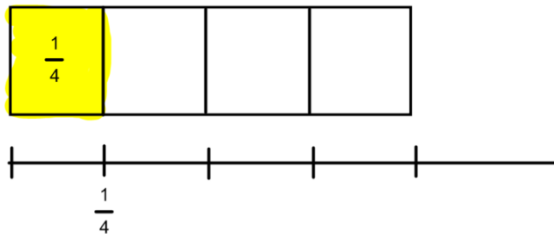
Student Response Example	Indicators
	<p><i>Label points on number line accurately</i></p> <p><i>Student may extend number line past 4/4, using equal spaces and improper fractions until the end of the number line</i></p> <p><i>Student may measure the first equal part using a tool (possibly a thumb, scrap paper or fraction bar) and uses that same tool to measure the space between each partition</i></p> <p><i>Student model and written description show that they understand equal parts as defined by the space between 0/4 and 1/4</i></p>
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
<p>How far can you extend the number line continuing with fourths?</p> <p>Where would $\frac{1}{2}$ fall on this number line? What other equivalent fractions can you place on the number line?</p> <p>What fraction would fall between 0 and $\frac{1}{4}$ on the number line?</p>	<p>Plot improper fractions on a number line https://learnzillion.com/lessons/1730-plot-improper-fractions-on-a-number-line</p> <p>Identify equivalent fractions using a number line https://learnzillion.com/lessons/1732-identify-equivalent-fractions-using-a-number-line</p>

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Students will be able to plot fractions on a number line.
Content Standard(s):	<p>Developing an Understanding of Fractions as Numbers</p> <p>NF 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p style="padding-left: 40px;">a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.</p>
Targeted Practice Standard :	<p><u>MP7</u> Look for and make use of structure.</p> <p><u>MP6</u> Attend to precision</p>
Mathematical Goals	Success Criteria
<ul style="list-style-type: none"> • Understand fractions as numbers • Understand that the denominator indicates how many equal parts of the whole 	<ul style="list-style-type: none"> • Students will be able to partition a number line into equal parts as indicated by the denominators and plot the fractions within 0-1.
Launch (Probe and Build Background Knowledge)	
<p><i>Students will need to understand how to partition an area of a rectangle into equal parts. Draw a number line below the partitioned rectangle to show how the two relate.</i></p>	

First show a rectangle that is partitioned into four equal parts. Shade in one fourth. Ask the students to identify the fractional part that is shaded. Shade in two fourths, ask the students: What does the numerator represent? What does the denominator represent?

Next draw a number line directly below the partitioned rectangle. Exaggerate how you bring down the lines from the area model and place it on the number line. Ask the students; how are these two pictures connected?

Then have students volunteer to label the number line, they must explain their thinking as they show their work.



Instructional Task

Purpose: Today we are going to practice placing fractions on a number line.

Engage (Setting Up the Task)

What do you notice when we place fractions on the number lines? (page 15-16)

Place $0/2$, $1/2$ and $2/2$ on the number line.

Place $0/4$, $1/4$, $2/4$, $3/4$ and $4/4$ on the number line.

Place $0/8$, $1/8$, $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, $7/8$ and $8/8$ on the number line.

Explore (Solving the Task)

Students work independently for about 10 minutes and then compare work and discuss in pairs/ trios.

Students may not understand how many equal parts they need to partition the line into. Review vocabulary with them: What does the denominator tell us? What does the numerator tell us?

Students may struggle partitioning the number line into equal parts. Encourage them to use their fraction bars to help them. How can you use a fraction bar to help you? Suggest that when we partition into four equal pieces, it is easiest to split the number line in half first and then split each half in half to create fourths. To partition into eighths we split each fourth in half. Which fraction is easiest to plot first, why?

Elaborate (Discuss Task and Related Mathematical Concepts)

Bring students back together as a whole class. Post the common misunderstanding question on the board. Give students 2-3 minutes to turn and talk to a partner about the problem. Listen carefully to their discussions and choose a few students to share out with the class. Students should be able to explain that the number line is not partitioned into equal parts. After class discussion, give students time to revise their own work based on their new understanding.

Checking for Understanding

Purpose: This should be done while students are working independently or in their pair/trio discussion. Use the student work that they are working on and ask questions:

What do you think was the easiest fraction to plot first, why? Can you explain how you partitioned the number line? How are the fraction bar and number line related? Why did you plot $(1/4)$ on that point?

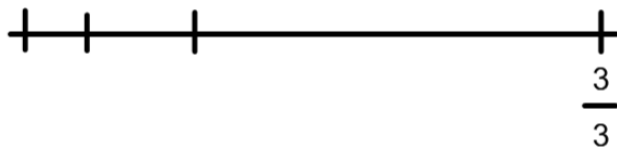
When first task is completed, students may play Number Line Roll game. (page 18)

Common Misunderstanding

Bob plotted $3/3$ on the number line.

Do you agree or disagree with him?

Explain your thinking.



Students may assume that the last point shown on a number line is always equal to 1 or in this case $3/3$. This affects their ability to partition the equal parts correctly. You may prompt the students by asking; Why do you think Bob placed $3/3$ at the end of the number line? Where should $3/3$ fall and why?

Checking for Understanding

Purpose: Students will complete an exit slip; work should be collected and analyzed by the teacher for next steps.

Your second grade friend wants to place $\frac{5}{6}$ on a number line. Use a picture and words to explain how to plot the fraction.

Closure

Purpose: *Students place a green, yellow or red dot at the top of the exit slip to indicate how they feel about their learning. Green to indicate they feel confident, yellow to indicate that they are on their way to understanding or red to indicate they are not confident.*

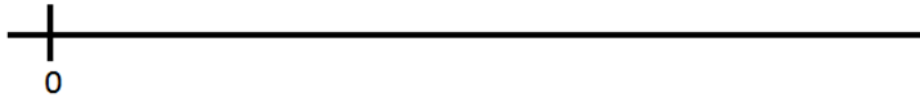
Use this information when planning for the next lesson. If you notice that a student got the problem incorrect but placed a green dot on their page. You will need to meet with them and discuss their misconceptions. If a student put a red dot on their exit slip and got the problem correct then they will need a little confidence boost.

Extension Task

Purpose: Students will place fractions with different denominators on the same number line. What do you notice when you place several fractions on the same number line? Students may use different colors to place the halves, fourths and eighths. They should draw over the color when placing an equivalent fraction to show that more than one fraction falls on the same point.

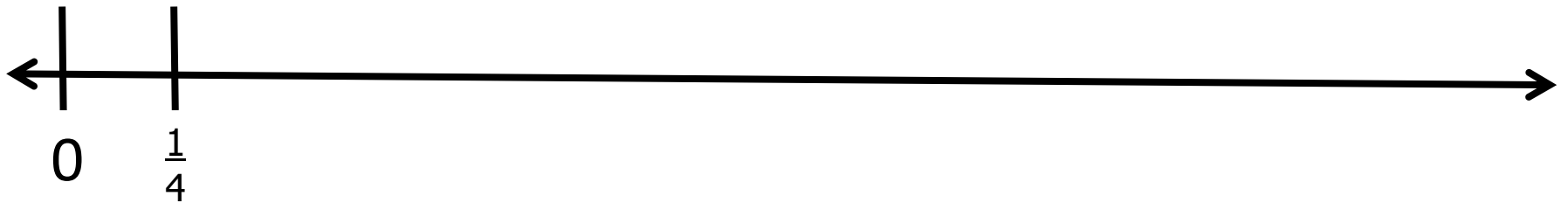
1. Place the following fractions on the number line.

$$\frac{1}{2}, \frac{1}{4}, \frac{4}{4}, \frac{1}{8}, \frac{2}{8}$$



Mathematical Checkpoint

Plot $\frac{4}{4}$ on the number line.



Explain to a friend how you knew where to put $\frac{4}{4}$.

Instructional Task

Place $0/2$, $1/2$ and $2/2$ on the number line.



Place $0/4$, $1/4$, $2/4$, $3/4$ and $4/4$ on the number line.



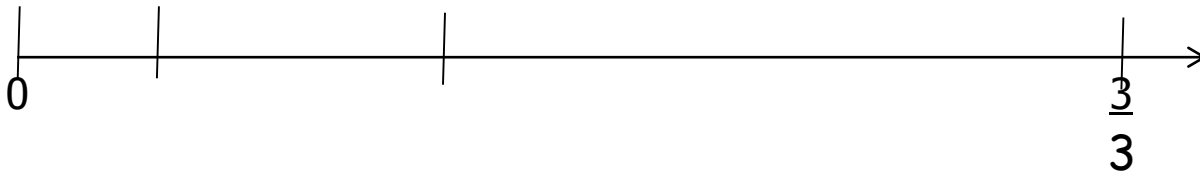
Place $0/8$, $1/8$, $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, $7/8$ and $8/8$ on the number line.



What do you notice about the number lines?

Common Misunderstanding

1. Bob plotted $\frac{3}{3}$ on the number line. Do you agree or disagree with him?
Explain your thinking.



Number Line Roll

Materials: ruler, dice

1. Draw a six inch number line that begins with 0 and ends with 1.



2. Roll a die. Divide your number line into this number of equal segments.



3. Label the segments as fractions and explain your reasoning.
4. Repeat until you have four different number lines.

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<http://www.k-5mathteachingresources.com/support-files/number-line-roll.pdf>

Check for Understanding – exit slip

Your second grade friend wants to place $\frac{5}{6}$ on a number line. Use a picture and words to explain how to plot the fraction.

Place a green, yellow or red dot at the top of the exit slip to indicate how they feel about their learning.


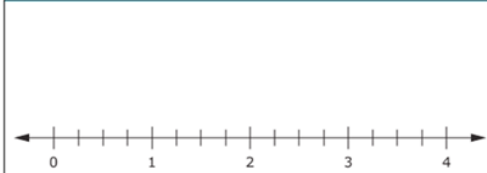
Green = confident Yellow = on my way to understanding Red = not confident

Extension Activity

1. Place the following fractions on the same number line.

$$\frac{1}{2}, \frac{1}{4}, \frac{4}{4}, \frac{1}{8}, \frac{2}{8}$$



Research and review of standard	
Content Standard(s):	Standard(s) for Mathematical Practice:
<p>Develop understanding of fractions as numbers NF 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p>	<p>MP6. Attend to precision</p> <p>MP7. Look for and make use of structure</p>
Smarter Balanced Claim	Smarter Balanced Item
<p>Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p>	<div style="border: 1px solid #ccc; padding: 10px;"> <div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="background-color: #00a0e3; color: white; padding: 2px 5px; font-weight: bold; margin-right: 10px;">15</div> <div style="margin-left: 10px;">  </div> </div> <p style="font-size: small; margin-bottom: 10px;">Drag each fraction to the correct location on the number line.</p> <div style="text-align: center; margin-bottom: 10px;">  </div> <div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> <div style="border: 1px solid #ccc; padding: 5px; text-align: center;">$\frac{5}{5}$</div> <div style="border: 1px solid #ccc; padding: 5px; text-align: center;">$\frac{1}{4}$</div> <div style="border: 1px solid #ccc; padding: 5px; text-align: center;">$\frac{4}{1}$</div> <div style="border: 1px solid #ccc; padding: 5px; text-align: center;">$\frac{2}{4}$</div> </div> <div style="border: 1px solid #ccc; height: 20px; width: 100%;"></div> </div>
CPR Pre-Requisites (Conceptual Understanding, Procedural Skills, and Representations)	Conceptual Understanding and Knowledge
	<ul style="list-style-type: none"> Understand fractions as numbers Understand fractions as numbers on a number line Recognize the unit fraction ($1/b$) created when an interval from 0-1 that has been partitioned into b equal parts Understand a unit fraction is the size part from 0 to $1/b$ on a number line Recognize a fraction a/b is composed of a lengths of $1/b$

	<p>Procedural Skills</p> <ul style="list-style-type: none"> Place fractions on a number line Partition a number line into an equal number of parts as indicated by the denominator <p>Representational</p> <ul style="list-style-type: none"> Represent fractions on a number line <p>Mathematical Connections</p> <ul style="list-style-type: none"> Know that on a number line, numbers to the left are smaller and numbers to the right are larger <p>Know the numerator is the top term in a fraction and refers to the number of parts counted and the denominator is the bottom term in a fraction and refers to the number of total parts in the whole</p>
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Standards Progression		
Grade(s) below	Target grade	Grade(s) above
<p><i>2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</i></p> <p><i>2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.</i></p>	<p><i>3.NF.A.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</i></p> <p><i>3.NF.A.2b Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</i></p> <p><i>3.NF.A.3a Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</i></p> <p><i>3.NF.A.3b Recognize and generate simple equivalent fractions, e.g.,</i></p>	<p><i>4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</i></p> <p><i>4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</i></p>

	$1/2=2/4$, $4/6=2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model	5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)
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Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- The number line is longer than needed, students may want to place one whole at the end of the number line

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Partitioning into equal parts as indicated by the denominator

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- When students plot whole numbers on a number line; the number of lines that are drawn are equal to the number of intervals. This is not true when partitioning a number line into fractional parts.
- Overgeneralizing the meaning of fractions due to limited experiences with dividing regions and sets into fair shares. For example, students identify a half as one of two parts, rather than one of two equal parts.