Mathematics Instructional Cycle Guide

Comparing and Ordering Fractions (4.NF.A.2)

Created by Tomasa Raver, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Extend understanding of fraction equivalence and ordering.

4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g. by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >,=, or <, and justify the conclusions, e.g. by using a visual fraction model.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP.1 Make sense of problems and persevere in solving them.

- Do students read problems carefully and use reasonable strategies, such as finding common denominators, using benchmark fractions, and proximity to 1 to accurately solve the problem?

MP.3 Construct viable arguments and critique the reasoning of others.

- Do students construct arguments using visual fraction models and number lines to explain why one fraction is greater or less than another fraction?

MP.6 Attend to precision.

- Do students follow necessary steps to carry out a strategy and check their work for accuracy when ordering fractions?

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(pages 2, 13)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(pages 3-6)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(pages 7-12)**
- Supporting lesson materials **(pages 13-29)**
- Precursory research and review of standard **4.NF.A.2** and assessment items that illustrate the standard **(pages 30-32)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the ***Comparing Fractions Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

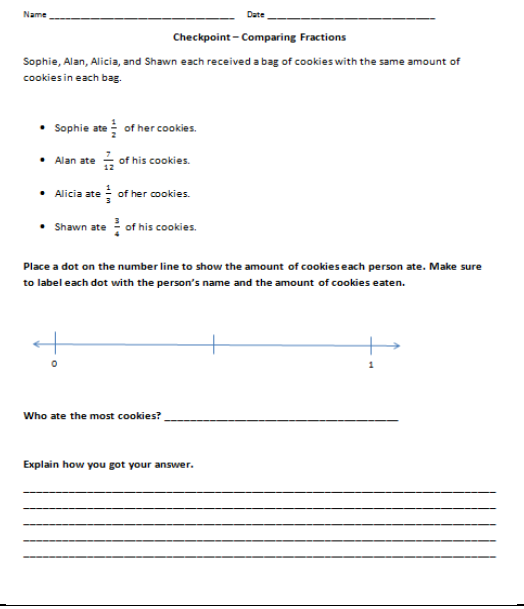
MATERIALS REQUIRED

- **Copies of handouts (one of each problem, five of each support handout, and a class set of checkpoints, probe, exit problem, extension task, and self-assessment)**
- **Markers**
- **Poster paper or Smart board and Document camera**

TIME NEEDED

Comparing Fractions Checkpoint administration**: 15 minutes**
Follow-Up Lesson Plan: **60 minutes**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2"> 
Name _____ Date _____
Checkpoint – Comparing Fractions
Sophie, Alan, Alicia, and Shawn each received a bag of cookies with the same amount of cookies in each bag.
• Sophie ate $\frac{1}{2}$ of her cookies.
• Alan ate $\frac{7}{12}$ of his cookies.
• Alicia ate $\frac{1}{3}$ of her cookies.
• Shawn ate $\frac{3}{4}$ of his cookies.
Place a dot on the number line to show the amount of cookies each person ate. Make sure to label each dot with the person's name and the amount of cookies eaten.
0 1
Who ate the most cookies? _____
Explain how you got your answer.
_____
_____
_____
_____
_____</td><td>CT Core Standard:</td><td>4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g. by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >,=, or <, and justify the conclusions, e.g. by using a visual fraction model.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>Can students compare and order fractions with unlike denominators?

Do students use a benchmark of ½ to identify fractions as greater or less than ½?

How do students partition and use a number line to help them compare fractions?</td></tr>
</table>

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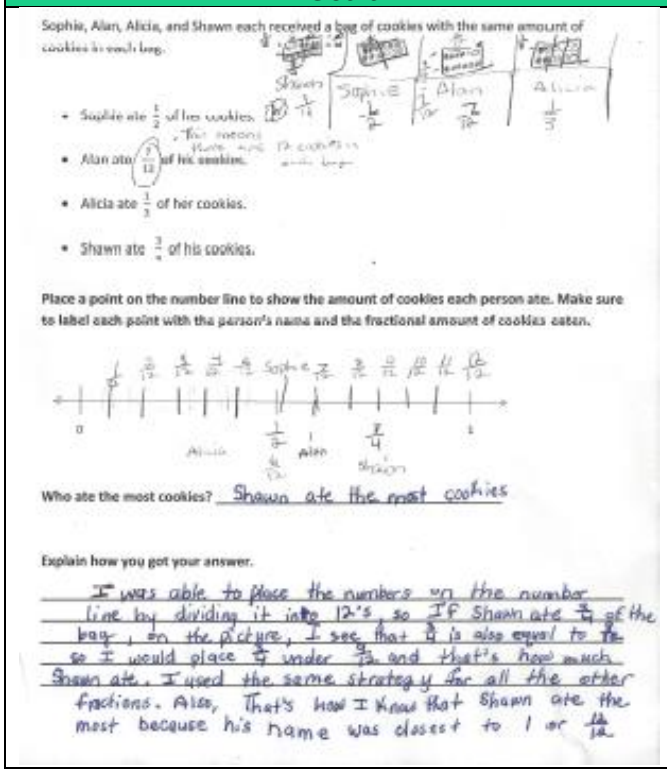
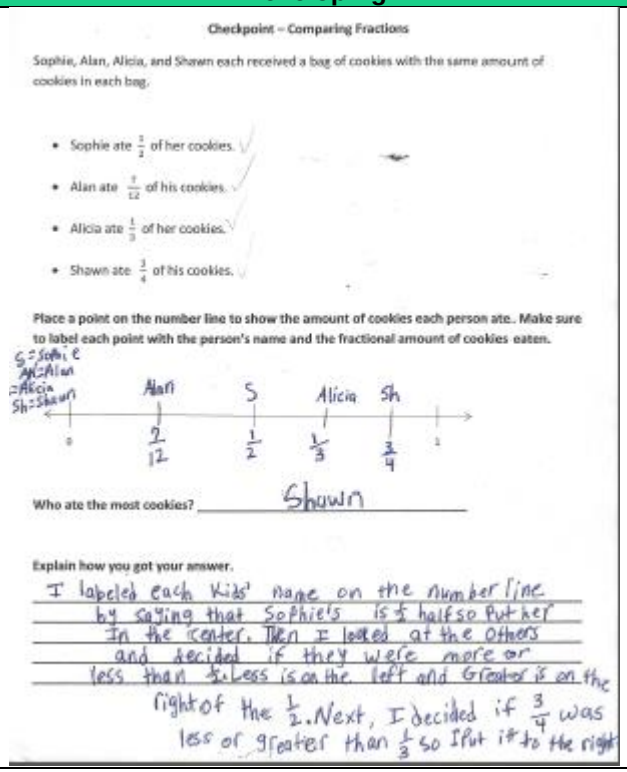
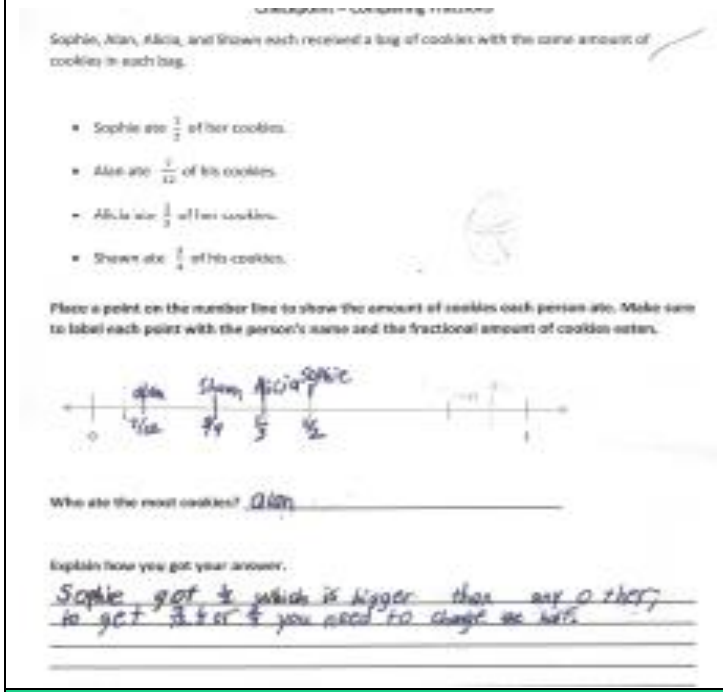
Step 2: Analyze and Interpret Student Work Student Response Guide		
Got It	**Developing**	**Getting Started**
		

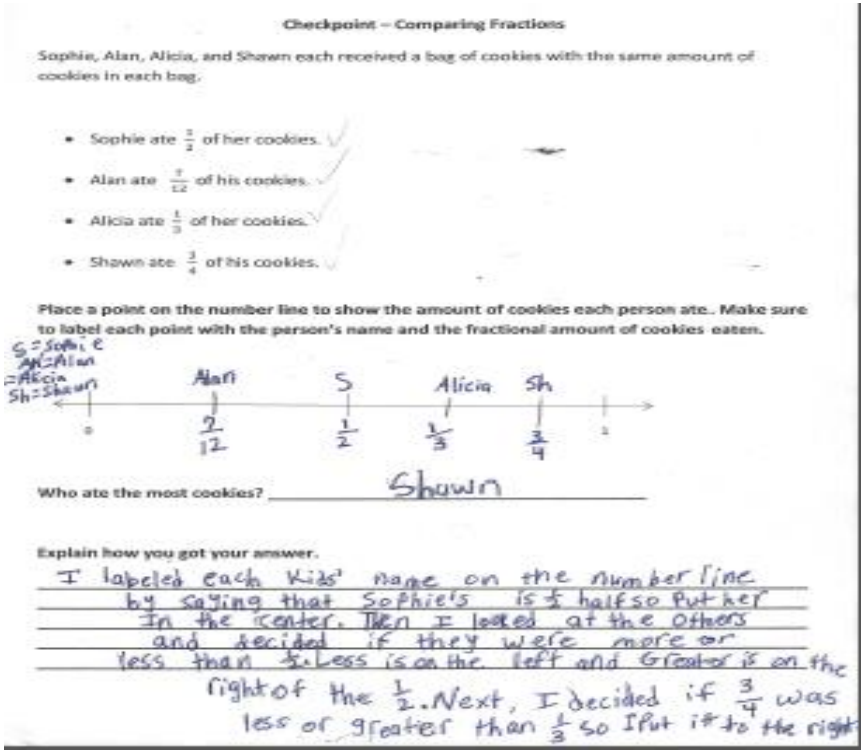
*Full size copies of student work are included with the support materials (pages 14-16)

LearnZillion

Getting Started	
Student Response Example	**Indicators**
	• Student work may reveal incomplete understanding or overgeneralizes whole number concepts to fraction work (i.e. the larger the denominator the greater the fraction) • Student does not understand that equivalent fractions represent the same number (i.e. $\frac{1}{2}$ and $\frac{6}{12}$ or $\frac{1}{3}$ and $\frac{4}{12}$) • Student does not consider the relationship between the numerator and denominator in order to determine the value of the fraction • Student is not able to use the number line to compare fractions and reason about their size.(i.e. fractions are incorrectly placed and labeled on the number line) .
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
Q: Tell me why you decided to place your fractions at each point. What makes fraction A larger than fraction B? Q: What does the numerator represent in each fraction? Q: What does the denominator represent in each fraction? Q: Show student visual fraction models of $\frac{1}{2}$ and $\frac{6}{12}$. What do you notice about these fractions? Then show a number line. Ask student to label $\frac{1}{2}$ on number line. Then show a $\frac{3}{4}$ visual fraction model. Ask students where they would place $\frac{3}{4}$ on the number line. How do you know it goes there? What is the relationship between $\frac{3}{4}$ and $\frac{1}{2}$?	http://learnzillion.com/lessons/844-compare-fractions-with-different-numerators-and-denominators-using-area-models http://learnzillion.com/lessons/1433-compare-fractions-with-different-denominators-using-number-lines http://learnzillion.com/lessons/843-compare-fractions-with-different-numerators-and-denominators-using-a-number-line Use fraction strips to show various fractions. Using the fraction strips, ask students to create and partition a number line in order to compare the fractions.

Developing

Student Response Example



Checkpoint – Comparing Fractions

Sophie, Alan, Alicia, and Shawn each received a bag of cookies with the same amount of cookies in each bag.

- Sophie ate $\frac{1}{2}$ of her cookies.
- Alan ate $\frac{7}{12}$ of his cookies.
- Alicia ate $\frac{1}{3}$ of her cookies.
- Shawn ate $\frac{3}{4}$ of his cookies.

Place a point on the number line to show the amount of cookies each person ate. Make sure to label each point with the person's name and the fractional amount of cookies eaten.

S=Sophie
Al=Alan
A=Alicia
Sh=Shawn

Alan $\frac{7}{12}$ S $\frac{1}{2}$ Alicia $\frac{1}{3}$ Sh $\frac{3}{4}$

Who ate the most cookies? Shawn

Explain how you got your answer.

I labeled each kids' name on the number line by saying that Sophie's is $\frac{1}{2}$ half so Put her in the center. Then I looked at the others and decided if they were more or less than $\frac{1}{2}$. Less is on the left and Greater is on the right of the $\frac{1}{2}$. Next, I decided if $\frac{3}{4}$ was less or greater than $\frac{1}{3}$ so I Put it to the right

Indicators

- Student reveals understanding that $\frac{1}{2}$ is midway between 0 and 1 by labeling the midpoint as $\frac{1}{2}$.
- Student explanation reveals some understanding of comparing fractions using a number line (i.e. student describes how they placed the fractions on the number line and then used the placement to help them determine which fraction was largest)
- The number line is not partitioned and labeled correctly
- Student may use faulty reasoning to explain how they determined who ate the most cookies (i.e. Alan ate the most because 7 is the largest numerator)

In the Moment Questions/Prompts

Q: Tell me why you decided to place your fractions at each point.

Q: How did the fraction $\frac{1}{2}$ help you place your other fractions?

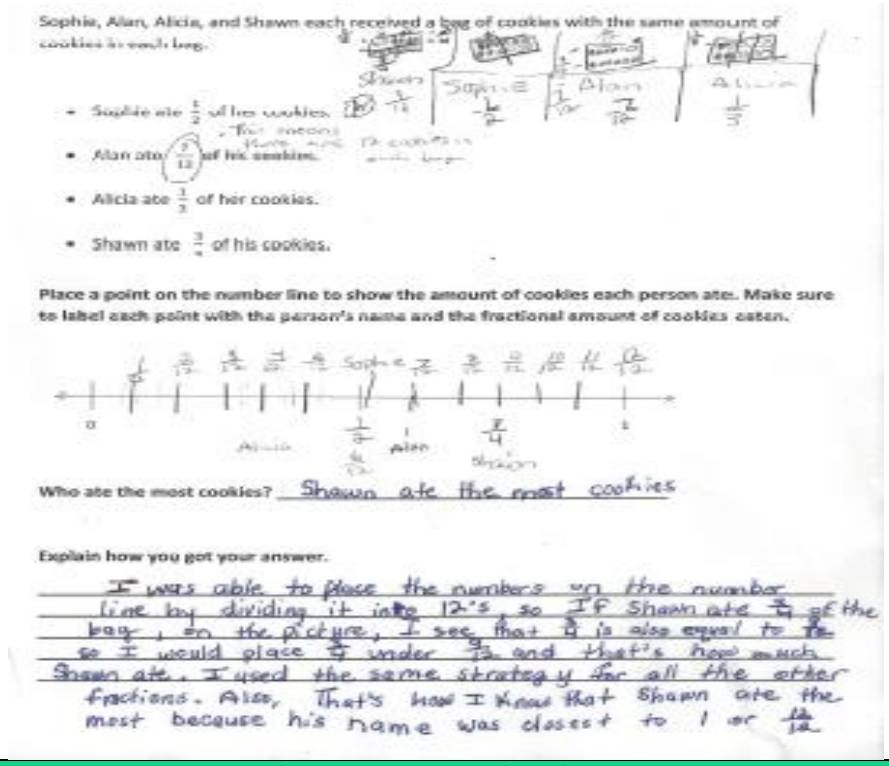
Q: How do you know that _____ ate the most cookies?

Q: Can you draw a model to prove that your answer is correct?

Closing the Loop (Interventions/Extensions)

http://learnzillion.com/lessons/845-create-common-denominators-using-number-lines

http://learnzillion.com/lessons/846-create-common-denominators-using-area-models

Got it	
Student Response Example	**Indicators**
	• Student accurately partitions a number line and places fractional amounts at correct locations • Student uses $\frac{1}{2}$ as a benchmark to compare fractions • Student uses appropriate reasoning to explain how he/she knows who ate the most cookies (location on number line, common denominators, benchmark fractions, closer to 1, etc.) • If visual models are used, they are accurately sized and support student reasoning about the problem
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
Q: Can you prove that Shawn ate the most cookies another way? Q: What patterns do you notice when comparing fractions? Q: How much more of his cookies would Alan have needed to eat in order to have eaten the most cookies?	Ask students to create a poster of the different strategies you can use to compare fractions. Provide examples for each strategy. When is it most efficient to use each strategy?

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Solve word problems that require comparing and ordering fractions with different numerators and denominators by using a reasonable strategy (common denominator, using benchmark of ½, etc.). Justify conclusion by using numbers, words, or visual models.
Content Standard(s):	**4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g. by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >,=, or <, and justify the conclusions, e.g. by using a visual fraction model.
Targeted Practice Standard :	**MP.1 Make sense of problems and persevere in solving them** • Do students read problems carefully and use reasonable strategies, such as finding common denominators, using benchmark fractions, and proximity to 1 to accurately solve the problem? **MP.3 Construct viable arguments and critique the reasoning of others** • Do students construct arguments using visual fraction models and number lines to explain why one fraction is greater or less than another fraction? **MP.6 Attend to precision** • Do students follow necessary steps to carry out a strategy and check their work for accuracy when ordering fractions?

Mathematical Goals	Success Criteria
• Understand that fractions are numbers and can be compared and ordered • Understand that fractions can be compared by using a variety of strategies	• Use a variety of strategies to compare fractions with unlike denominators • Read and solve word problems accurately • Use words, pictures, or numbers to justify solution

Launch (Probe and Build Background Knowledge)

Purpose: Access and build background knowledge of how visual fraction models and number lines can be used to compare and order fractions. Start the lesson by posing the following problem:

Use the symbols <, >, or = to compare the following fractions. You may use any of the strategies we learned in order to help you compare.

$\frac{3}{5} \square \frac{3}{8}$ $\frac{4}{6} \square \frac{2}{6}$ $\frac{2}{3} \square \frac{7}{12}$

Be prepared to discuss which strategy you used to solve the problem.

Instructional Task

Purpose: In groups, students will solve a problem requiring them to compare and order fractions with unlike denominators by using any reasonable strategy. Students will justify their answer by using words, numbers, and visual models to explain their thinking.

We have been comparing and ordering fractions using visual models, $\frac{1}{2}$ as a benchmark, or finding equivalent fractions to determine whether a fraction is less than or greater than another fraction. Today, you are going to choose the most efficient strategy to help you solve everyday problems.

Engage (Setting Up the Task)

Divide the class into five groups. Using a cooperative learning model, assign each student a role (i.e., reader, reporter, recorder, task manager). Make sure that each student understands that this role is in conjunction with being an active participant, contributing his/her ideas toward solving the assigned problem. Explain that they will work together as a group to solve the problem given to them. Each group has a different problem to solve. To demonstrate their learning, they will present their problem and solution to the class, explaining their reasoning and strategies used for solving the problem.

Introduce the Task

Provide each group with a problem to solve. (See handout attachments)

Provide each student with a whiteboard and marker.

*Provide each group with poster board and markers, if needed.

Provide each group with the geoboard , fraction bars, and *Finding Common Denominator handouts* to help them construct and compare fractions, if needed. Any other manipulatives you deem appropriate and have used in your class to teach fractions may also be used.

After the problem is read aloud in each group, allow 3-5 minutes of independent work time for each student to solve the problem using their whiteboards. At the end of this time, groups may begin their discussions and work together to solve the problem. Call time after about 15 minutes (give more time if needed).

*Depending on the technology available in your classroom, each group presents the problem and solution to the class using a document camera, Smart Board, or on poster paper, if technology is not available. A class discussion will follow each presentation.

If groups finish early, they may work on the extension provided at the bottom of the page.

Example of a problem:

Tracy, Luke, and Carter have entered a Walk-A –Thon. They must each raise $100. Tracy has raised $\frac{1}{4}$ of her money. Luke has raised $\frac{4}{5}$ of his money. Carter has raised $\frac{7}{10}$ of his money. Carter says that he has raised the most money so far. Is he correct? Can you prove or disprove his statement? On the handout provided or on poster paper:

- Use an efficient strategy to solve the problem
- Explain why you think you are correct

*Remember to discuss your thinking as a group while solving the problem. Also, as a group, decide how you will explain your reasoning to the class before the scribe writes it down on the paper or poster board. You and your

group should be in agreement on how the problem is solved. If you disagree, you need to prove your position to your group.

Extending the Task:
Challenge students who are ready to figure out how much money each person has raised. How much more money do they need to raise to meet their goal?

Explore (Solving the Task)

The teacher provides each group with one of the five word problems (see handouts).
Students may use the geoboard handout, fraction bars handout, or the *Finding A Common Denominator* handout to help them solve the problem. They may also use any other reasonable method to help them accurately compare fractions.

Teacher will circulate while groups work together to solve the problem.
Groups must show their work by using numbers, words, or visual models to justify their solution.

Scaffold Questions/ Prompts:

- What do you know about fractions to help you compare them?
- What strategy do you think is the most efficient to help you compare your fractions?
- What is the least common multiple for these denominators?
- How do you know this fraction is equivalent to _____?
- Can you draw a model to prove your thinking?

Advancing Questions/Prompts:

- How can you prove that this fraction is equivalent to _____?
- How can you find a common denominator for all three denominators?
- Do you have to find a common denominator for all three denominators, or is there another way to compare and order the fractions?
- Can you prove your thinking in a different way?

Elaborate (Discuss Task and Related Mathematical Concepts)
Groups will share their problem and solution using the document camera and Smartboard technology (if available). If not, groups may copy problem, write solution, and explanation on poster paper to be shared with the class. Class discussion will be facilitated to elicit evidence of student understanding.

Possible Discussion Questions:
What strategies did you find most helpful to help you compare the fractions? Which strategy is most efficient in this example? How do you know?

Did two people in your group have the same answer but a different approach to solving the problem?

Did listening to each other's explanations help you to better understand the problem? How was it helpful or not helpful?

What have you learned about using equivalent fractions to compare/order fractions?

What have you learned about comparing and ordering fractions?

Would placing the fractions on a number line be helpful in answering the question?

Checking for Understanding

Purpose: Use a challenging fraction comparison problem to encourage different methods of comparison, and not the use of only one strategy, to solve a problem.

- Write the following fractions on the whiteboard. Allow students to order the fractions and explain their reasoning by using paper or whiteboards.

$$\frac{4}{9}, \quad \frac{2}{8}, \quad \frac{8}{10}$$

- Students turn and talk to their partner about how they ordered their numbers, explaining step-by-step their reasoning for ordering them in that way.

- Facilitate a discussion where students share their responses. Make a list of the various strategies used by students. Emphasize that more than one strategy may be used at one time to compare and order the fractions. Make sure that you encourage multiple approaches to solving the problem.

- If no student uses finding a common numerator, demonstrate this technique. The common numerator is 8.

So, $\frac{4 \; x \; 2}{9 \; x \; 2} = \frac{8}{18}$

$\frac{2 \; x \; 4}{8 \; x \; 4} = \frac{8}{32}$

$\frac{8}{10} = \frac{8}{10}$

Now, students can use their knowledge about numerators and denominators to make their comparisons. Since the numerator is the same, we know that the greater the denominator the smaller the number.

So, we know that

$$\frac{8}{32} < \frac{8}{18} < \frac{8}{10}$$

Therefore,

$$\frac{2}{8} < \frac{4}{9} < \frac{8}{10}$$

Note to teacher: By working through some of the different techniques for comparing fractions, students will develop a deeper number sense of fractions. If all suggested methods given below do not come up in the discussion, the teacher could make a suggestion and let the students work further. One of the main objectives of this task is to help students develop a strategic sense of when to use which technique or method.

Students may:
- Use visual models
- Compare to benchmarks (0, ½ and 1)
- Find a common denominator
- Find a common numerator
- Use logical reasoning skills about numerators and denominators

Common Misunderstanding

Purpose: Check for student interpretation of a fraction as a number and that the student is not relying on whole number ideas and concepts to compare fractions.

Monica and Lisa are reading the same book. Monica has read $\frac{5}{12}$ of her book and Lisa has read $\frac{2}{3}$ of her book. Monica says she has read more because five is greater than two. Lisa says that Monica is wrong, that she has read the most so far. Who is correct? How do you know?

Use words, pictures, or numbers to justify your answer.

Checking for Understanding

Purpose: Check student understanding of comparing and ordering fractions.

**This Exit Ticket can be used at the end of this lesson or used to launch the next lesson.*

Pass out the following "Exit Ticket" to students. Instruct students to solve the problem and explain their thinking.

Compare the following fractions. Order them by using a number line. Explain your reasoning.

$\frac{1}{2}$ $\frac{10}{12}$ $\frac{3}{8}$

Explain your reasoning below:

_____
_____
_____
_____

What's another strategy you could use to compare these fractions?

_____
_____
_____
_____

Closure

Purpose: Provide students an opportunity to monitor and reflect on their own understanding of comparing and ordering fractions with unlike denominators by using the following self-assessment.

Think about your learning. Place a check in the box that best matches your level of success with each item.

I can accurately solve problems that require comparing fractions with unlike denominators.	I can do this with help.	I can do this by myself.	I can do this on my own and explain to someone else
I can explain if one fraction is <, >, or = to another fraction by using a variety of strategies.	I can do this with help.	I can do this by myself.	I can do this on my own and explain to someone else

One thing I feel I learned well is

_____
_____
_____

One thing I feel I still need more time learning is

_____
_____
_____

Extension Task

Purpose: Students demonstrating a strong understanding of comparing and ordering fractions can be challenged with the following extension task.

Think about the word problems we solved in our last lesson. Write a word problem for your partner to solve where he/she will need to compare and order fractions with unlike denominators. Avoid using fractions that can be easily compared (i.e. $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$). Instead, use fractions that challenge your partner.

Make sure to solve the problem yourself and be able to discuss the solution with your partner.

Name _____ Date _____

Checkpoint – Comparing Fractions

Sophie, Alan, Alicia, and Shawn each received a bag of cookies with the same amount of cookies in each bag.

- Sophie ate $\frac{1}{2}$ of her cookies.
- Alan ate $\frac{7}{12}$ of his cookies.
- Alicia ate $\frac{1}{3}$ of her cookies.
- Shawn ate $\frac{3}{4}$ of his cookies.

Place a point on the number line to show the amount of cookies each person ate. Make sure to label each point with the person's name and the fractional amount of cookies eaten.

Who ate the most cookies? _____

Explain how you got your answer.

_____
_____
_____
_____
_____

Getting Started

Checkpoint – Comparing Fractions

Sophie, Alan, Alicia, and Shawn each received a bag of cookies with the same amount of cookies in each bag.

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- Alan ate $\frac{7}{12}$ of his cookies.
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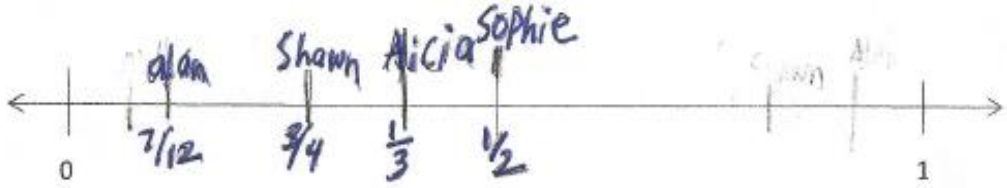

Who ate the most cookies? alan

Explain how you got your answer.

Sophie got ½ which is bigger than any other, to get 7/12, ⅓ or ¾ you need to change the half.

Developing

Sophie, Alan, Alicia, and Shawn each received a bag of cookies with the same amount of cookies in each bag.

- Sophie ate $\frac{1}{2}$ of her cookies.
- Alan ate $\frac{7}{12}$ of his cookies.
- Alicia ate $\frac{1}{3}$ of her cookies.
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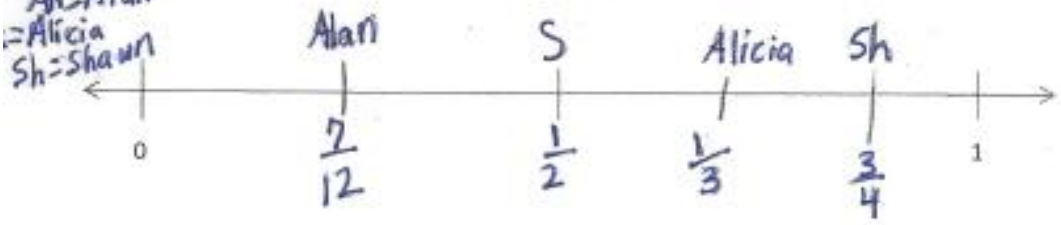
Place a point on the number line to show the amount of cookies each person ate. Make sure to label each point with the person's name and the fractional amount of cookies eaten.

S=Sophie
AN=Alan
=Alicia
Sh=Shawn



Who ate the most cookies? Shawn

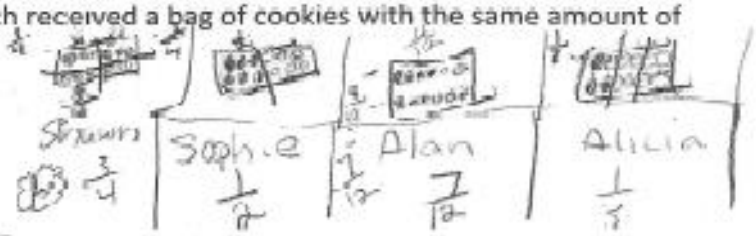
Explain how you got your answer.

I labeled each kids' name on the number line by saying that Sophie's is $\frac{1}{2}$ half so Put her In the center. Then I looked at the others and decided if they were more or less than $\frac{1}{2}$. Less is on the left and Greater is on the right of the $\frac{1}{2}$. Next, I decided if $\frac{3}{4}$ was less or greater than $\frac{1}{3}$ so I Put it to the right

Got It

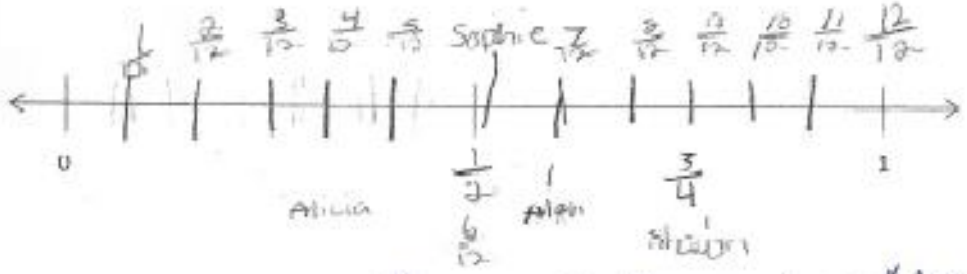
Checkpoint – Comparing Fractions

Sophie, Alan, Alicia, and Shawn each received a bag of cookies with the same amount of cookies in each bag.



- Sophie ate $\frac{1}{2}$ of her cookies.
- Alan ate $\frac{7}{12}$ of his cookies. this means there are 12 cookies in each bag
- Alicia ate $\frac{1}{3}$ of her cookies.
- Shawn ate $\frac{1}{4}$ of his cookies.

Place a point on the number line to show the amount of cookies each person ate. Make sure to label each point with the person's name and the fractional amount of cookies eaten.



Who ate the most cookies? Shawn ate the most cookies

Explain how you got your answer.

I was able to place the numbers on the number line by dividing it into 12's, so If Shawn ate $\frac{3}{4}$ of the bag, on the picture, I see that $\frac{3}{4}$ is also equal to $\frac{9}{12}$ so I would place $\frac{3}{4}$ under $\frac{9}{12}$ and that's how much Shawn ate. I used the same strategy for all the other fractions. Also, That's how I know that Shawn ate the most because his name was closest to 1 or $\frac{12}{12}$

Lesson: **Solving Problems that Require Comparing and Ordering Fractions**

Probe: Launching the Lesson and Building Background Knowledge

Use the symbols <, >, or = to compare the following fractions. You may use any of the strategies we learned in order to help you compare.

$\frac{3}{5} \square \frac{3}{8}$ $\frac{4}{6} \square \frac{2}{6}$ $\frac{2}{3} \square \frac{7}{12}$

Be prepared to discuss which strategy you used to solve the problem.

Directions to teacher: Make copies of the following problems. Give each group a different problem to solve.

PROBLEM #1

Tracy, Luke, and Carter have entered a Walk-A-Thon. They must each raise $100. Tracy has raised $\frac{1}{4}$ of her money. Luke has raised $\frac{4}{5}$ of his money. Carter has raised $\frac{7}{10}$ of his money. Carter says that he has raised the most money so far. Is he correct? Can you prove or disprove his statement?

- Use any reasonable strategy to solve the problem
  (i.e., visual models, use of number line, use ½ as benchmark, find common denominators, etc.).

- Explain why you think you are correct.
  *Remember to discuss your thinking as a group while solving the problem. Also, as a group, decide how you will explain your reasoning to the class before the scribe writes it down on the paper or poster board. You and your group should be in agreement on how the problem is solved. If you disagree, you need to prove your position to your group.

Show work.

Explain your answer.

_____

_____

_____

_____

_____

_____

Extension: How much money has each student raised? How much more money does each student need to raise to meet their goal?

Problem #2

Mary, Levi, and Sarah are sharing a large pizza. Mary ate $\frac{2}{8}$ of the pizza, Levi ate $\frac{1}{4}$, and Sarah ate $\frac{4}{16}$. Sarah says she ate the most pizza. Is she correct? Can you prove or disprove her statement?

- Use any reasonable strategy to solve the problem (i.e., visual models, use of number line, use ½ as benchmark, find common denominators, etc.).
- Explain why you think you are correct.

*Remember to discuss your thinking as a group while solving the problem. Also, as a group, decide how you will explain your reasoning to the class before the scribe writes it down on the paper or poster board. You and your group should be in agreement on how the problem is solved. If you disagree, you need to prove your position to your group.

Show work.

Explain your answer.

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Extension: What fraction of the pizza did they eat altogether? What fraction of the pizza is left?

Problem #3

Crystal, Jimmy, and Chloe are running a 20 mile race. Crystal has run $\frac{1}{2}$ the distance, Jimmy has run $\frac{3}{5}$ the distance, and Chloe has run $\frac{3}{4}$ of the distance. Who is in the lead? Who has run the least? Can you prove how you are correct?

- Use any reasonable strategy to solve the problem
  (i.e., visual models, use of number line, use ½ as benchmark, find common denominators, etc.).
- Explain why you think you are correct.

*Remember to discuss your thinking as a group while solving the problem. Also, as a group, decide how you will explain your reasoning to the class before the scribe writes it down on the paper or poster board. You and your group should be in agreement on how the problem is solved. If you disagree, you need to prove your position to your group.

Show work.

Explain your answer.

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Extension: How much farther does the person in the lead need to run to finish the race? How many miles has the person in the lead run? How many more miles are needed to finish the race?

Problem #4

Lisa, Jessie, and Chris are collecting cans as a fundraiser for their school. They need to collect a total of 200 cans. Lisa has collected $\frac{2}{5}$ of the cans, Jessie has collected $\frac{1}{4}$ of the cans, and Chris has collected $\frac{3}{10}$ of the cans. Who has collected the most cans? Who has collected the least? Can you prove how you are correct?

- Use any reasonable strategy to solve the problem (i.e., visual models, use of number line, use ½ as benchmark, find common denominators, etc.).
- Explain why you think you are correct.

*Remember to discuss your thinking as a group while solving the problem. Also, as a group, decide how you will explain your reasoning to the class before the scribe writes it down on the paper or poster board. You and your group should be in agreement on how the problem is solved. If you disagree, you need to prove your position to your group.

Show work.

Explain your answer.

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Extension: What fraction of the cans do they still need to collect? How many cans have they collected? How many cans do they still need to collect?

Problem #5

Sandy, Jarrell, and Molly are painting a room. Sandy has painted $\frac{1}{3}$ of the room, Jarrell has painted $\frac{2}{8}$ of the room, and Molly has painted $\frac{3}{8}$ of the room. Sandy says she has painted the most. Is she correct? Can you prove or disprove her statement?

- Use any reasonable strategy to solve the problem
  (i.e., visual models, use of number line, use ½ as benchmark, find common denominators, etc.).
- Explain why you think you are correct.

*Remember to discuss your thinking as a group while solving the problem. Also, as a group, decide how you will explain your reasoning to the class before the scribe writes it down on the paper or poster board. You and your group should be in agreement on how the problem is solved. If you disagree, you need to prove your position to your group.

Show work.

Explain your answer.

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Extension: Who has painted the least? How much more of the room is left to paint?

Name_____ **Date**_____

Check for Understanding

(Common Misunderstanding)
Comparing and Ordering Fractions

Monica and Lisa are reading the same book. Monica has read $\frac{5}{12}$ of her book and Lisa has read $\frac{2}{3}$ of her book. Monica says she has read more because five is greater than two. Lisa says that Monica is wrong, that she has read the most so far. Who is correct? How do you know?

Who is correct, Lisa or Monica? _____

Explain your thinking. Use words, pictures, or numbers to justify your answer.

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Comparing Fractions by Finding

A Common Denominator

The following fractions may be difficult to compare by using manipulatives or logical reasoning about what we know about fractions. To make sure we are comparing correctly, we sometimes need to find a common denominator.

$$\frac{5}{8} \qquad \frac{7}{10}$$

Step 1: Make a number set of the multiples for each denominator. Find the least common multiple.

8: 8, 16, 24, 32, 40, 48

10: 10, 20, 30, 40, 50

Step 2: Once you have found the least common multiple, find equivalent fractions for each fraction.

$$\frac{5 \times 5}{8 \times 5} = \frac{25}{40}$$

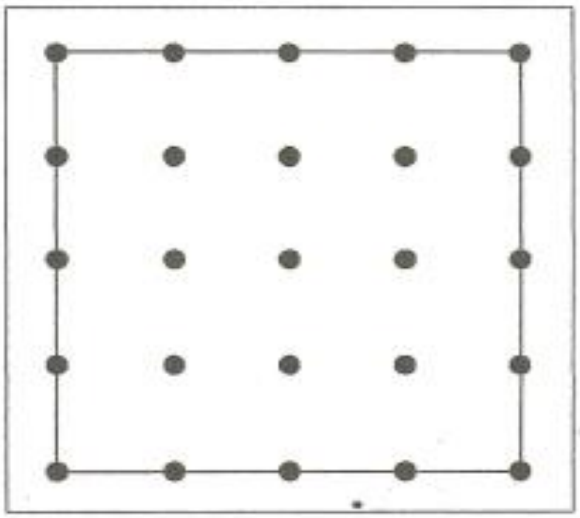
$$\frac{7 \times 4}{10 \times 4} = \frac{28}{40}$$

Step 3: Now, you can compare your fractions by using the equivalent fractions.

$$\frac{25}{40} < \frac{28}{40}$$

Therefore, $\frac{5}{8} < \frac{7}{10}$

Geoboard Area of One



Area = 1 sq. unit

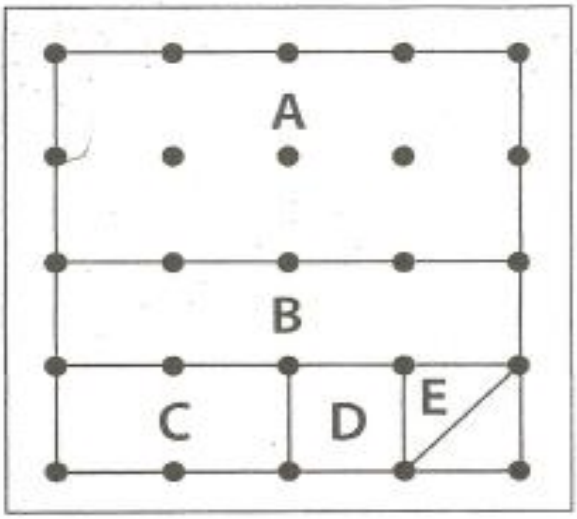
A = 1/2

B = 1/4

C = 1/8

D = 1/16

E = 1/32



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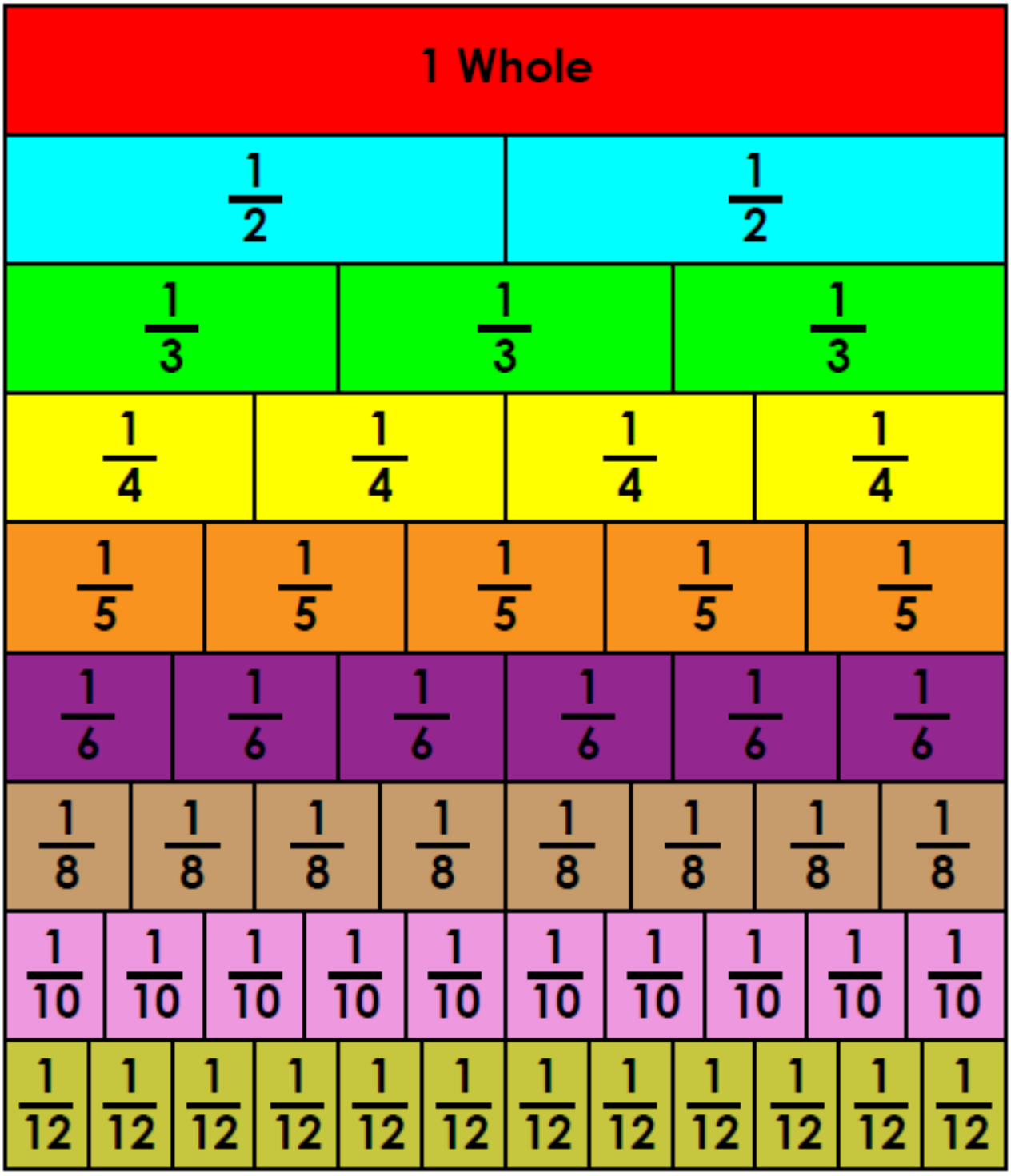
Fraction Strips



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Name _____ **Date** _____

Extension Task

Think about the word problems we solved in our last lesson. Write a word problem for your partner to solve where he/she will need to compare and order fractions with unlike denominators. Avoid using fractions that can be easily compared (i.e. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$). Instead, use fractions that challenge your partner.

Make sure to solve the problem yourself on another sheet of paper and be able to discuss the solution with your partner.

Word Problem:

Partner's Name:
Solution (include explanation):

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Name_____ **Date**_____

Exit Ticket

Comparing and Ordering Fractions

Compare the following fractions. Order them by using a number line. Explain your reasoning.

$\frac{1}{2}$ $\frac{10}{12}$ $\frac{3}{8}$

Explain your reasoning below:

_____

_____

_____

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_____

What's another strategy you could use to compare these fractions?

_____

_____

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Name _____ **Date** _____

Self - Assessment

Think about your learning. Place a check in the box that best matches your level of success with each item.

I can accurately solve problems that require comparing fractions with unlike denominators.	I can do this with help.	I can do this by myself.	I can do this on my own and explain to someone else
I can explain if one fraction is <, >, or = to another fraction by using a variety of strategies.	I can do this with help.	I can do this by myself.	I can do this on my own and explain to someone else

One thing I feel I learned well is:

_____

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One thing I feel I still need more time learning is:

_____

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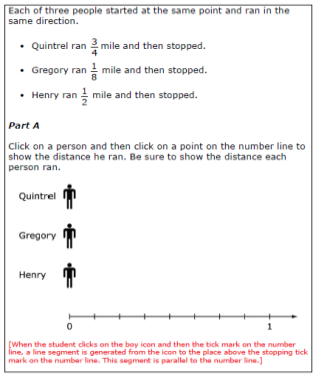
Research and review of standard	
Content Standard(s):	**Standard(s) for Mathematical Practice:**
Extend understanding of fraction equivalence and ordering. **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g. by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >,=, or <, and justify the conclusions, e.g. by using a visual fraction model.	**MP.1 Make sense of problems and persevere in solving them.** **MP.3 Construct viable arguments and critique the reasoning of others.** **MP.6 Attend to precision.**
Smarter Balanced Claim	***Smarter Balanced Item***
Claim 1: Concepts and procedures *Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.*	 *Grade 4 Mathematics Sample TE Item C1 TF Part A*

CPR Pre-Requisites
(Conceptual Understanding, Procedural Skills, and Representations)

Conceptual Understanding and Knowledge
- Understand fractions as numbers
- Understand fractions as numbers on a number line (a fraction is the size part from 0 to $\frac{a}{b}$ on a number line)
- Understand equivalent fractions represent the same part of a whole or the same point on the number line.
- Understand that fractions can be compared using a variety of strategies
- Understand that fractions can only be compared when the wholes are the same.

Procedural Skills
- Identify/place fractions on a number line
- Compare fractions with like denominators

Representational
- Represent fractions using a variety of models (fraction strips, number lines, etc.)

Social knowledge
- Know the numerator is the top term in a fraction and refers to the number of parts counted
- Know the denominator is the bottom term in a fraction and refers to the number of total parts in a whole
- Know equivalent means equal or the same size
- Know that “benchmark” means a point of reference to which other fractions may be compared (such as 0, ½ and 1)

Standards Progression		
Grade(s) below	**Target grade**	**Grade(s) above**
3.NF.2.a Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. 3.NF.2.b Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths of 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. 3.NF.3.a Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. 3.NF.3.b Recognize and generate simple equivalent fractions, (e.g., $^1/_2 = {}^2/_4$, $^4/_6 = {}^2/_3$). Explain why the fractions are equivalent (e.g., by using a visual fraction model). 3.NF.3.c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. 3.NF.3.d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <, =, or >, and justify the conclusions, e.g., by using a visual fraction model.	4.NF.1 Explain why a fraction *a*/*b* is equivalent to a fraction (*n* x*a*)/(*n* x *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- The use of unlike denominators may be challenging to some students
- The number line is partitioned into eight equal units
- Students may not understand the race context

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students may look at the denominator as whole numbers and disregard the numerator, thinking $\frac{1}{8}$ is greater than $\frac{1}{2}$
- Students may not identify $\frac{1}{2}$ being the same as $\frac{2}{4}$ and $\frac{4}{8}$ on the number line
- Students may know that $\frac{3}{4}$ is to the right of $\frac{1}{2}$ on the number line but not recognize that $\frac{3}{4}$ is the same as $\frac{6}{8}$ (equivalent fractions represent the same point on the number line)
- Students may count the number of tic marks rather than the intervals between the tic marks

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- Students may believe that $\frac{1}{2}$ is in the middle of the number line and place it there without counting the number of tick marks or intervals in the whole
- Students may treat the numerator and denominator as isolated numerals instead of interpreting the fraction as a number