TEACHFEST

CONNECTICUT SUMMER ACADEMY

TeachFest Guide

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Mathematics

CONNECTICUT

STATE DEPARTMENT OF EDUCATION

Table of Contents

Criteria for a Worthwhile Task

Effective mathematics instruction engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow for multiple entry points and varied solution strategies.[1] The tasks teachers select to engage students in studying mathematics have a significant impact on students' opportunities to learn and their perceptions about what mathematics is. This document lists some of the key criteria that make an instructional task worthwhile.

What is a Worthwhile Task?

"A worthwhile task is a project, question, problem, construction, application, or exercise that engages students to reason about mathematical ideas, make connections, solve problems, and develop mathematical skills."

- National Council for Teachers of Mathematics

A Worthwhile Task…

- Is directed at essential mathematical content addressed by the grade level standards and is aligned to the lesson objective.
- Makes connections between concepts and procedures.
- Makes connections between different mathematical topics.
- Requires reasoning (non-algorithmic), higher-level thinking, and problem solving.
- Connects to real-world or interesting mathematical situations that are familiar to students.
- Is appropriately challenging and accessible (engages students' interests and intellect).
- Provides multiple ways to demonstrate understanding of mathematics concepts and procedures.
- Has various solutions or can be approached by students in multiple ways using different solution strategies.
- Requires students to illustrate or explain mathematical ideas.
- Encourages student engagement and discourse.
- Connects to other important mathematical ideas.
- Empowers students to unravel their misconceptions.
- Provides an opportunity to practice important skills.

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[1] *Leinwand, S., Brahier, D., and Huinker, D., Principles to Action*, 2014, pg. 17

Other Sources:
The NCTM Brief *Why is Teaching with Problem-Solving Important to Student Learning* (April 8, 2010); Schrock, C. Norris, Norris, K., Pugalee, D., et. al, *NCSM Great Tasks for Mathematics* (2013)

Common Core State Standards Shifts in Mathematics

1. **Focus** strongly where the Standards focus

Focus: The Standards call for a greater focus in mathematics. Rather than racing to cover topics in a mile-wide, inch-deep curriculum, the Standards require us to significantly narrow and deepen the way time and energy is spent in the math classroom. We focus deeply on the major work* of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom.

2. **Coherence**: **think** across grades, and **link** to major topics within grades

Thinking across grades: The Standards are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.

Linking to major topics: Instead of allowing additional or supporting topics to detract from the focus of the grade, these concepts serve the grade level focus. For example, instead of data displays as an end in themselves, they are an opportunity to do grade-level word problems.

3. **Rigor**: in major topics* pursue:
   - **conceptual understanding**,
   - procedural skill and **fluency**, and
   - **application** with equal intensity.

Conceptual understanding: The Standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

Procedural skill and fluency: The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions such as single-digit multiplication so that they have access to more complex concepts and procedures.

Application: The Standards call for students to use math flexibly for applications in problem-solving contexts. In content areas outside of math, particularly science, students are given the opportunity to use math to make meaning of and access content.

High-level Summary of Major Work in Grades K–8

K–2	Addition and subtraction—concepts, skills, and problem solving; place value
3–5	Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving
6	Ratios and proportional relationships; early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear algebra and linear functions

*For a list of major, additional and supporting clusters by grade, please refer to 'Focus in Math' at achievethecore.org/focus pp. 4–12

K-12 Standards for Mathematical Practice (SMP)

1. **Make sense of problems and persevere in solving them.**

   Mathematically proficient students: Seek and explain the meaning of the problem. Look for efficient ways to represent and solve it. Ask themselves "Does this make sense?" and "Can I solve the problem in a different way?" Identify the connections between two different approaches to a problem.

2. **Reason abstractly and quantitatively.**

   Mathematically proficient students: Decontextualize – to manipulate symbolic representations by applying properties of operations. Contextualize – to understand the meaning of the number or variable as related to the problem. Understand the meaning of the quantities, not just how to compute them.

3. **Construct viable arguments and critique the reasoning of others.**

   Mathematically proficient students: Construct arguments using verbal or written explanations (expressions, equations, graphs, etc.) Evaluate their own thinking and the thinking of others by asking questions like, "How did you get that?" "Why is that true?" "Does that always work?"

4. **Model with mathematics.**

   Mathematically proficient students: Model problem situations symbolically, graphically, and contextually. Connect and explain the connections between different representations. Use all the different representations as appropriate to a problem context. Modeling is defined as the act of constructing a mathematical representation of a situation (not a noun).

5. **Use appropriate tools strategically.**

   Mathematically proficient students: are sufficiently familiar with appropriate tools to decide when each tool is helpful, knowing both the benefit and limitations. They can detect possible errors and identify relevant external mathematical resources, and use them to pose or solve problems.

6. **Attend to precision.**

   Mathematically proficient students: Use clear and precise language/definitions in their discussions with others and in their own reasoning. State the meaning of symbols. Specify units of measure**.** Calculate accurately and efficiently with an appropriate degree of precision for the problem context.

7. **Look for and make use of structure.**

   Mathematically proficient students: Look for patterns or structures to model and solve problems. Example – Addition is the same, no matter which number system is used; to add, one must add like units. This practice is about making observations.

8. **Look for and express regularity in repeated reasoning.**

   Mathematically proficient students: Use repeated reasoning to: understand algorithms, make generalizations about patterns, derive formulas and evaluate the reasonableness of intermediate results. This practice is about "doing" (performing operations).

EQuIP Rubric for Lessons & Units: Mathematics

Grade: **Mathematics Lesson/Unit Title:** **Overall Rating:**

I. Alignment to the Depth of the CCSS	II. Key Shifts in the CCSS	III. Instructional Supports	IV. Assessment
The lesson/unit aligns with the letter and spirit of the CCSS: ○ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning. ○ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed. ○ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.	*The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:* ○ **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades. ○ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions. ○ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following: – **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations. – **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding. – **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	*The lesson/unit is responsive to varied student learning needs:* ○ Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media. ○ Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline. ○ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking. ○ Addresses instructional expectations and is easy to understand and use. ○ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners. – Supports diverse cultural and linguistic backgrounds, interests and styles. – Provides extra supports for students working below grade level. – Provides extensions for students with high interest or working above grade level. *<u>A unit or longer lesson should:</u>* ○ Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share). ○ Gradually remove supports, requiring students to demonstrate their mathematical understanding independently. ○ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time. ○ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	*The lesson/unit regularly assesses whether students are mastering standards-based content and skills:* ○ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS. ○ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts. ○ Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance. *<u>A unit or longer lesson should:</u>* ○ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.
Rating: 3 2 1 0	**Rating: 3 2 1 0**	**Rating: 3 2 1 0**	**Rating: 3 2 1 0**

EQuIP Rubric for Lessons & Units: Mathematics

Directions: The Quality Review Rubric provides criteria to determine the quality and alignment of lessons and units to the Common Core State Standards (CCSS) in order to: (1) Identify exemplars/ models for teachers' use within and across states; (2) provide constructive criteria-based feedback to developers; and (3) review existing instructional materials to determine what revisions are needed.

Step 1 – Review Materials

- Record the grade and title of the lesson/unit on the recording form.
- Scan to see what the lesson/unit contains and how it is organized.
- Read key materials related to instruction, assessment and teacher guidance.
- Study and work the task that serves as the centerpiece for the lesson/unit, analyzing the content and mathematical practices the tasks require.

Step 2 – Apply Criteria in Dimension I: Alignment

- Identify the grade-level CCSS that the lesson/unit targets.
- Closely examine the materials through the "lens" of each criterion.
- Individually check each criterion for which clear and substantial evidence is found.
- Identify and record input on specific improvements that might be made to meet criteria or strengthen alignment.
- Enter your rating 0 – 3 for Dimension I: Alignment.

Note: Dimension I is non-negotiable. In order for the review to continue, a rating of 2 or 3 is required. If the review is discontinued, consider general feedback that might be given to developers/teachers regarding next steps.

Step 3 – Apply Criteria in Dimensions II – IV

- Closely examine the lesson/unit through the "lens" of each criterion.
- Record comments on criteria met, improvements needed and then rate 0 – 3.

When working in a group, individuals may choose to compare ratings after each dimension or delay conversation until each person has rated and recorded their input for the remaining Dimensions II – IV.

Step 4 – Apply an Overall Rating and Provide Summary Comments

- Review ratings for Dimensions I – IV adding/clarifying comments as needed.
- Write summary comments for your overall rating on your recording sheet.
- Total dimension ratings and record overall rating E, E/I, R, N – adjust as necessary.

If working in a group, individuals should record their overall rating prior to conversation.

Step 5 – Compare Overall Ratings and Determine Next Steps

- Note the evidence cited to arrive at final ratings, summary comments and similarities and differences among raters. Recommend next steps for the lesson/unit and provide recommendations for improvement and/or ratings to developers/teachers.

Additional Guidance on Dimension II: Shifts - When considering *Focus* it is important that lessons or units targeting additional and supporting clusters are sufficiently brief – this ensures that students will spend the strong majority of the year on major work of the grade. See the *K-8 Publishers Criteria for the Common Core State Standards in Mathematics*, particularly pages 8-9 for further information on the focus criterion with respect to major work of the grade at www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf. With respect to *Coherence* it is important that the learning objectives are linked to CCSS cluster headings (see www.corestandards.org/Math).

Rating Scales

Rating for Dimension I: Alignment is non-negotiable and requires a rating of 2 or 3. If rating is 0 or 1 then the review does not continue.

Rating Scale for Dimensions I, II, III, IV:	Overall Rating for the Lesson/Unit:
3: Meets most to all of the criteria in the dimension **2**: Meets many of the criteria in the dimension	**E:** Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV **(total 11 – 12)** **E/I:** Exemplar *if* Improved – Aligned and needs some improvement in one or more dimensions **(total 8 – 10)**
1: Meets some of the criteria in the dimension **0**: Does not meet the criteria in the dimension	**R:** Revision Needed – Aligned partially and needs significant revision in one or more dimensions **(total 3 – 7)** **N:** Not Ready to Review – Not aligned and does not meet criteria **(total 0 – 2)**

Descriptors for Dimensions I, II, III, IV:	Descriptor for Overall Ratings:
3: **Exemplifies CCSS Quality -** meets the standard described by criteria in the dimension, as explained in criterion-based observations. **2**: **Approaching CCSS Quality -** meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.	**E: Exemplifies CCSS Quality –** Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric. **E/I: Approaching CCSS Quality –** Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.
1: **Developing toward CCSS Quality -** needs significant revision, as suggested in criterion-based observations. **0**: **Not representing CCSS Quality -** does not address the criteria in the dimension.	**R: Developing toward CCSS Quality –** Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others. **N: Not representing CCSS Quality** – Not aligned and does not address criteria.

Part A: Example Task Research

Name: Andrea Smith

Grade: 5	**Task Title:** Banana Pudding **Source**: https://www.illustrativemathematics.org/illustrations/1196	
Domain & Cluster	**Content Standard(s)**	**Mathematical Practice(s)**
NF: Number and Operations---Fractions Cluster: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	5.NF.B.7.B Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Shifts of the Common Core State Standards		
Focus Find your grade here**.**	**Coherence** Wiring Document Learning Trajectories http://www.corestandards.org/	**Rigor** Select all that apply
Major Supporting Additional	Builds from: 3.OA.2, 4.NF.4 Connects to: 5.NF.4, 5.NF.6 Builds up to: 6.NS.1	**Conceptual Understanding** Procedural Fluency **Application**

Part B: Example Task Analysis

The purpose of the Task Analysis tool is to support teachers in selecting worthwhile tasks. While a task may not meet every Criteria of a Worthwhile Task, teachers should use their judgment to determine if the task meets enough of the criteria to an acceptable level in its current form to be a useful instructional task, or should be improved to better meet specific criteria.

Part B

Task Analysis		
Criteria of Worthwhile Task	**Rating**	**Notes on how to enhance or improve the task**
1. Is grade-level appropriate *Does the task align to the grade-level standard?*	1 2 3 **4**	
2. Makes connections between concept and procedures *What conceptual understandings are embedded in this task that students should takeaway as a result of doing this task?* *Does the task support students in understanding the concept(s) upon which a procedure is based?* *What misunderstandings or roadblocks may be surfaced by the task?*	1 2 3 **4**	Students may not make the connection that finding the number of quarter cups needed requires fraction division. This debrief will explain this connection explicitly.
3. Makes connections between different mathematical topics *What other cluster(s) or standard(s) does the task directly connect or potentially connect to?*	1 **2** 3 4	Students may approach the task pictorially or use fraction sense instead of connecting the task to fraction division. Explore adding a component to develop the relationship between fraction multiplication and division or develop debrief questions to support making this connection.
4. Requires reasoning (nonalgorithmic thinking) *Does the task require students to do more than just reproduce a procedure?* *What misunderstandings or roadblocks may be surfaced by the task?*	1 2 3 **4**	Because students may not connect the task with division (or any mathematical operation), keep the equation component to support making a mathematical connection.
5. Connects to real situations that are familiar and relevant to students *Does the task connect mathematical concepts and procedures to their real world applications?* *What contextual features of the task must the students understand in order to successfully engage in the task?*	1 2 **3** 4	Select a recipe students may want to make or be familiar with and add a component that requires students to reason about how using different measuring cup sizes would affect the number of cups needed for each ingredient. Possibly use quarters and halves to develop patterns between halves and fourths.
6. Is appropriately challenging and accessible (engages students' interests and intellect) *What modifications or accommodations may need to be in place to support learning by all students (e.g., ELLs, students w/ IEPs or 504s as well as students whose understanding is beyond the task)?*	1 2 **3** 4	Extend the task by adding a part that requires students to consider how changing the cup size (divisor) would change the solution or how doubling the recipe would change the solution.
7. Provides multiple ways to demonstrate understanding of the mathematical concepts and procedures *How might students solve the problem? What prior knowledge might they apply to the task?* *Is there more than one approach students could take to solve the task? Is there more than one solution to the task?*	1 2 **3** 4	Specify that students can use words, pictures, models, and/or numbers to represent their solutions. In the task debrief, be sure to select multiple representations including pictures, number lines, and equations to illustrate the number of cups

		needed for each ingredient.
8. Requires students to illustrate or explain mathematical ideas *What representations could be used to model the mathematical concepts and procedures embedded in this task?* *How will students explain or justify their thinking?*	1 2 3 **4**	Possibly add a component that requires students to explain why their solutions are reasonable.

Part C: Example Task Rewrite

Created by:	Andrea Smith
Task Title	Oreo Dirt Pudding Task
Grade:	5
Standard:	5.NF.B.7b
Original Task:	https://www.illustrativemathematics.org/illustrations/1196

Rewritten or revised task

Oreo Dirt Pudding Recipe

3 cups Oreo cookies (crushed)
2 cups of milk
1 cup chocolate pudding mix
1 ½ cups Cool Whip

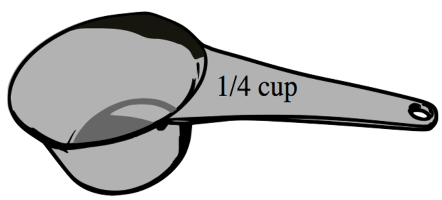
Optional: Gummy Worms

Yields 8 cups

http://www.ohnuts.com/blog/dirt-pudding-cups-with-gummy-worms-recipe/

You are making the Oreo Dirt Pudding recipe. You decide to double the recipe so you will have more pudding to share.

You realize you only have a quarter cup in your kitchen to use.



a.) For each ingredient, find how many quarter cups you would need for the doubled recipe. Draw a picture or a model to prove your solutions.

b.) For each ingredient, write an equation to show how many quarter cups you would need for the doubled recipe. Explain why your solution for each equation is reasonable.

c.) Now, you have a half cup instead of a quarter cup. Your friend claims you will need half as many cups for each ingredient. Is your friend's claim true? Explain why or why not using numbers, words, or pictures.

d.) You scoop out a half cup of Oreo Dirt Pudding to taste test. If you share it equally with a friend, how much pudding will each of you get? Write an equation that represents this situation.

Task extensions

e.) You have 12 cups of Oreo Dirt Pudding left. You decide to share the leftover pudding with your class. If you make $\frac{2}{3}$ cup servings of the pudding, will you have enough pudding left to share with 20 people? Use words, numbers, or pictures to prove your solution.

f.) Explain how the number of cups you would need for each ingredient would change if you used each of the measuring cups listed below to make the recipe.

$\frac{1}{3}$ cup $\frac{3}{4}$ cup

Part A: Task Research Template

Name:

Grade:		**Task Title:** **Source**:
Domain & Cluster	**Content Standard(s)**	**Mathematical Practice(s)**
Domain: **Cluster:**		1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Shifts of the Common Core State Standards		
Focus Find your grade here.	**Coherence** Wiring Document Learning Trajectories http://www.corestandards.org/	**Rigor** *Circle all that apply*
Major Supporting Additional	Builds from…. Connects to… Builds up to…	Conceptual Understanding Procedural Fluency Application

Part B: Task Analysis Template

The purpose of the Task Analysis tool is to support teachers in selecting worthwhile tasks. While a task may not meet every Criteria of a Worthwhile Task, teachers should use their judgment to determine if the task meets enough of the criteria to be a useful instructional task or if the task should be improved to better meet specific criteria.

Task Analysis		
Criteria of Worthwhile Task	**Rating**	**Notes on how to enhance or improve the task**
1. Is grade-level appropriate *Does the task align to the grade-level standard?*	1 2 3 4	
2. Makes connections between concept and procedures *What conceptual understandings are embedded in this task that students should takeaway as a result of doing this task?* *Does the task support students in understanding the concept(s) upon which a procedure is based?* *What misunderstandings or roadblocks may be surfaced by the task?*	1 2 3 4	
3. Makes connections between different mathematical topics *What other cluster(s) or standard(s) does the task directly connect or potentially connect to?*	1 2 3 4	
4. Requires reasoning (nonalgorithmic thinking) *Does the task require students to do more than just reproduce a procedure?* *What misunderstandings or roadblocks may be surfaced by the task?*	1 2 3 4	
5. Connects to real situations that are familiar and relevant to students *Does the task connect mathematical concepts and procedures to their real world applications?* *What contextual features of the task must the students understand in order to successfully engage in the task?*	1 2 3 4	
6. Is appropriately challenging and accessible (engages students' interests and intellect) *What modifications or accommodations may need to be in place to support learning by all students (e.g., ELLs, students w/ IEPs or 504s as well as students whose understanding is beyond the task)?*	1 2 3 4	
7. Provides multiple ways to demonstrate understanding of the mathematical concepts and procedures *How might students solve the problem? What prior knowledge might they apply to the task?* *Is there more than one approach students could take to solve the task? Is there more than one solution to the task?*	1 2 3 4	
8. Requires students to illustrate or explain mathematical ideas *What representations could be used to model the mathematical concepts and procedures embedded in this task?* *How will students explain or justify their thinking?*	1 2 3 4	

Adapted from Bay-Williams, J.M. McGatha, M., Kobbet, B., & Wray, J. (2014). *Mathematics Coaching: Resources and Tools for Coaches and Leaders, K-12.* Boston: Pearson

1 = The quality in the task is not evident, or it is not possible to address this quality with the task
2 = The quality is evident in minor ways or incorporating it is possible.
3 = The quality is evident in the task
4 = The quality is central to the task and is important to the success of the lesson.

Part C: Task Rewrite Template

Created by:	
Task Title	
Grade:	
Standard:	
Original Task:	

Rewritten or revised task
Questions to think about … • *What features of the original task do you like?* • *What is the mathematical content of the original problem?* • *What aspects of the original task make the students think and struggle?* • *What can be taken out or modified to create constructive struggling?* • *What steps does the original task give the students that they could come up with on their own?* • *Which of the 8 Standards for Mathematical Practice does the original task contain?* • *What can we modify to make sure that the Standards for Mathematical Practice are included?* • *Are the numbers in the original task purposeful, or could you change them to serve a specific purpose?* • *What features of the original task could be changed or improved?* • *What features could be added to the original task?* • *What features could be deleted from the original task?* • *How could you open up the original task so that there are multiple approaches or solutions?* • *Is there a real-world context you could use that would give students a reason to solve this original task?* • *How can you incorporate a feature that requires students to illustrate or explain their thinking?*
Task extensions

Considerations for Selecting Worthwhile Instructional Tasks

Selecting an instructional task for your lesson is no easy feat! Use the questions outlined below to jumpstart your thinking and set goals for the instructional outcomes of your task. You may not need to answer all of these questions when selecting a task, but reading through them or answering a few may help you get started or get back on track if you get stuck.

Questions to consider when selecting tasks

Questions about your instructional goals…

- What grade-level cluster(s) and content standard(s) do you want to target?
- What are the important mathematical ideas that you hope students will take from this task?
- Are there any misconceptions or common errors that you want students to confront or unravel?
- What mathematical representations or models do you want students use or explore?
- What procedural skills or fluencies do you want students to use or practice?
- Are you targeting any particular Standards for Mathematical Practice?
- How much time do you have?
- What materials/resources do you have at your disposal?

Questions about tasks that you find…

- Is the task directed at essential mathematical content addressed by the grade level cluster and standard?
- Does this task address the depth of the standard or a portion of the standard?
- Will the task contribute to the conceptual development of students and help students make sense of mathematics?
- Is the task open-ended, whether in answer or approach?
- Will the task engage students in a real-world mathematical problem?
- Does the task require students to illustrate or explain their mathematical ideas?
- Will the task encourage student engagement and discourse?
- Will the task require more than the application of facts and procedures, and encourage students to make connections and generalizations between concepts and procedures?
- Will the task empower students to unravel potential misconceptions?

Sources for Math Tasks

Illustrative Mathematics	www.illustrativemathematics.org *Illustrative Mathematics provides mathematical tasks, task solutions, and commentary on how the tasks illustrate content standards. The site also provides a fractions progression module as wells as videos and vignettes illustrating the Mathematical Practices.*
NCTM Illuminations	www.illuminations.nctm.org *Illuminations is a project designed by NCTM. The site has 600 lesson plans and over 100 activities including manipulatives, applets, and games. Lessons and activities are searchable by content standard.*
Mathalicious	www.mathalicious.com *Mathalicious provides middle school and high school teachers with lessons that help them teach math in a way that engages their students–in a way that helps students explore the math behind real world topics. Each Mathalicious lesson contains information on which content standards are covered in the lesson as well as the **Mathematical Practice Standards** Lessons address several standards to address more math in less time. The site offers some lessons for free and offers a pay-what- you-can pay subscription option for individual teachers to access the full library of lessons.*
Yummy Math	www.yummymath.com *Yummy Math provides teachers and students with mathematical tasks relevant to our world today. The site has a collection of tasks for grades 3-High school searchable by domain and by content standards. Tasks can be downloaded for free, and task solutions, teacher tips, and relevant attachments are accessible with a $16 annual membership fee.*
Balanced Assessment	http://balancedassessment.concord.org *The Balanced Assessment in Mathematics Program was developed by the Harvard Graduate School of Education. The site has a library of over 300 innovative mathematics assessment tasks for grades K to 12, available at no cost.*
Mathematics Assessment Project	http://map.mathshell.org/materials/index.php *The Mathematics Assessment Project provides formative assessment lessons focused on developing math concepts and non-routine problem solving. The lessons are designed to make student knowledge and reasoning visible, and help teachers to guide students in how to improve and monitor their progress.*
Dan Meyer's 3-Act Tasks	http://bit.ly/1w6jMqH *Dan Meyer has created a spreadsheet on which he has listed the Three Acts of many math tasks addressing high school and some middle school content standards. In this spreadsheet, a common question is posed based on Act One, and then Acts Two and Three are based on that question. The tasks and related materials are available for free.*

M. Lynn Breyfogle and Lauren E. Williams

Designing and Implementing Worthwhile Tasks

Teachers often need to alter mathematical tasks that they find in their district-adopted set of curriculum materials or develop new ones if none is present on a particular topic. However, how to best go about this work is not always clear. How do you make effective decisions about alterations? What should you keep in mind as you consider developing tasks to help your students with a particular idea or misconception? These and other questions were central in our minds as we developed a task to help students learn about elapsed time.

In preparation for the Pennsylvania System of School Assessment (PSSA) in mathematics, a fourth-grade class of twenty students in rural central Pennsylvania regularly used a practice book that covered a variety of mathematical concepts with a format similar to the state assessment. While observing this class, a student teacher noticed that many students exhibited frustration and a limited understanding of elapsed-time calculations. Because elapsed-time calculation is identified as a core standard in Pennsylvania (PDE 2008), teachers were concerned about this observation and therefore decided to facilitate a lesson that developed the concept of elapsed time in a meaningful way for the students.

Teaching elapsed time integrates easily at a point in a chapter that discusses regrouping in both addition and subtraction. Often it is taught at this point to encourage students to make the connection with time and extend the connection by using a number system other than base ten. Frequently, though, what happens is that students mistakenly assume that time *is* a base-ten system and arrive at erroneous answers. For example, given the problem, "Find how much time has elapsed between 9:33 a.m. and 11:08 a.m.," students follow a procedure of subtraction with regrouping and arrive at the nonsensical answer below:

$$\begin{array}{r} \cancel{1}1^{0}:{}^{1}08 \\ -\ \ 9:33 \\ \hline 1:75 \end{array}$$

Or, instead of approaching the problem by focusing on regrouping, teachers show a particular approach, such as missing-addend or counting-up strategies, similar to counting back change as a cashier does. Given the same problem, the solution procedure might be "from 9:33 a.m. to 10 a.m. is twenty-seven minutes, from 10 to 11 is one hour, and then eight more minutes, giving us an answer of one hour, thirty-five minutes." Although this approach has potential for helping students find elapsed time more successfully, the procedure pushes students to use a particular method that may or may not allow them to make sense of elapsed time.

Given the fact that the textbooks used in this classroom focused only on one particular procedure, the teachers decided to design a different set of tasks. This article describes the tasks they designed, discusses how the tasks aligned with NCTM's 1991 recommendations for worthwhile tasks, and further explains the importance of implementation and reflection strategies in helping students retain the challenging nature of the tasks (using the context of the fourth-grade elapsed-time lesson).

Creating Tasks

One of the most important and yet difficult aspects of designing a lesson is choosing or creating the worthwhile mathematical task a teacher wants students to engage in (Lappan and Friel 1993). The teacher must consider how well the task provides the opportunity for students to investigate the mathematics content in an open but structured way and how well the task

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connects with students' existing knowledge while pushing them deeper. When considering the design of a worthwhile task for elapsed time, Breyfogle and Williams kept this in mind by using the following questions as guideposts for the design:

- What goal is this to serve?
- Does it allow my students to make connections to content they already know?
- Does it allow for multiple solution methods or approaches?
- Does it encourage students to reason about mathematics and allow for them to communicate mathematically?

In the end, they arrived at a task requiring students to create their own schedule of a school day. Each student was to complete a blank schedule (see **table 1**) by arranging the eight provided classroom activities (e.g., math, reading, science) and assigning the starting and ending times for each. The students were also given specific parameters their schedule must meet (see **table 2**). They chose these parameters to increase students' cognitive demands by forcing them to go beyond simplistic solutions, such as using only whole-hour or half-hour increments.

For the second part of the task, students were placed into heterogeneous groups and instructed to discuss one another's schedules and choose one schedule to copy onto a poster for future reference. Students were to evaluate one another's schedules using the parameter list as a checklist to be sure the schedule met the criteria. Then they chose the schedule with the greatest variety of activity lengths. Finally, as a small group, they were to answer six analysis questions (see **fig. 1**) that included a comparison to a schedule the teacher had created with varied starting and ending times. As a whole group, students then had the opportunity to share their selected schedules and describe how they arrived at them as well as how they responded to the analysis questions.

What makes a task worthwhile?

A worthwhile task is a project, question, problem, construction, application, or exercise that engages students to reason about mathematical ideas, make connections, solve problems, and develop mathematical skills (NCTM 1991, pp. 24–25. On the basis of this definition, this task is worthwhile in that it (1) allows for connections, (2) incorporates multiple approaches and solutions, (3) requires higher-level thinking, and (4) facilitates reasoning and communication.

Table 1

Student Worksheet for Part 1 of Task

New School Schedule Name ______________________

Activity	Start Time	Stop Time
	9:00 a.m.	
		2:42 p.m.

*** Reminder**

- Lunch starts at 12:12 and is 32 minutes long.
- You must have gym, recess, science, math, reading, social studies, and art. You must spend at least ten minutes on each activity and at most one hour and ten minutes.

Table 2

Parameters for Part 1 of Task

Parameters for New School Day Schedule	
1.	The day must start at 9:00 a.m. and end at 2:42 p.m.
2.	Lunch must occur from 12:12 to 12:42.
3.	All eight daily activities must be used.
4.	Activities must last for at least ten minutes and no more than one hour and ten minutes.

1. A worthwhile task allows for connections. One identified aspect of a challenging task is its ability to connect to students' previous knowledge, experiences, and interests. The challenge presented in the elapsed-time lesson succeeds in connecting to the learners by capitalizing on a real-world experience (e.g., following a school schedule) familiar to each of the students, therefore making the prompt accessible to all the students in the class. The task also capitalizes on a common student desire to change the school schedule. In addition to connecting with students' experiences and interests, the task makes connections between elapsed time and prior knowledge, such as addition, subtraction, time telling, and more-than and less-than concepts.

2. A worthwhile task incorporates multiple approaches and solutions. A second identified attribute of a worthwhile task is that it can be approached in more than one way or has more than one legitimate answer. This attribute generates a shift in focus from the importance of providing the "correct answer" to an emphasis on the problem-solving process and the development of associated skills. Additionally, because multiple approaches offer flexibility for students with varied prior knowledge and experiences to engage with the task at their own level, differentiation is conveniently built into the task and fosters equity in the instruction.

The open-ended format of the new school schedule task allows for numerous student approaches and solutions. The activity could result in the class creating twenty different, yet legitimate, school schedules. It also prompts discussion of the different approaches and solutions to the same challenge with the potential for students to share numerous methods they used to complete the task. For example, one student could compute how many hours are available in the entire day, subtract minutes for lunch, and then split her time among the activities as equally as possible. Another student could recognize the parameter of a specific lunch time as a benchmark to split her thinking. She could start at lunch and work backward to the beginning of the day and then work from lunch forward to the end of the day. A third student could begin by giving each of her favorite activities one hour and ten minutes and then divvying up the remaining time to the other activities. Each of these different approaches uses slightly different thinking. It is important for teachers to consider the different methods their students could use.

3. A worthwhile task requires higher-level thinking. A third attribute of a worthwhile task is its demand for students to use higher-level thinking. The elapsed-time task effectively uses limiting parameters to increase cognitive demands. Limiting the length of time spent on daily activities (at least ten minutes and at most one hour, ten minutes) causes students to reconsider the duration of school subjects. For example, when this task was implemented, one student asked, "If I spend an hour and ten minutes on all of my favorite subjects, I won't have time for social studies. What should I do?" which then prompted a useful class discussion on how to resolve this problem. The time restrictions, coupled with the constraint of a designated lunch time increases the use of higher-order thinking skills as students reevaluate their schedules against the parameters.

4. A worthwhile task facilitates reasoning and communicating mathematically The last key attribute is that the task ought to facilitate the need for students to reason and communicate mathematically. Including tasks that provide opportunities for valuable classroom discussion is therefore critical. The second part of the elapsed-time challenge provides an opportunity for students to collectively reason through analysis questions (see **fig. 1**), communicate their ideas with group members, and then share them with the entire class. The point of this whole-group discussion is for students to have the opportunity to publicize their thinking about the situation's mathematics—not why they chose how much time to assign to art but rather how they thought about the concept of elapsed time and determined the mathematics of the schedule.

Figure 1

Student groups received questions during part 2 of their task.

Group Questions

1. Which group member's schedule had the class members spend the least amount of time on science? How much time did they spend?
2. Which group member's schedule had the class members spend the most amount of time on science? How much time did they spend?
3. Did any member's schedule have the class spend more time in gym than in reading? If so, whose? How much more time?
4. Did any group member's schedule have the class spend less time in math than Ms. Williams's schedule? If so, whose? How much less time?
5. Look at Ms. Williams's starting time for art and your starting time for art. Which group member's schedule started art class at a time farthest away from Ms. Williams's starting time for art? How much time elapsed between those two starting times?
6. Look at Ms. Williams's ending time for reading and your ending time for reading. Which group member's schedule ended reading at a time farthest away from Ms. Williams's ending time for reading? How much time elapsed between those two ending times?

Implementing the Elapsed-Time Task

After Williams briefly led the entire class through scheduling and planning a party for her elevenyear-old niece as an example, students independently created their own school-day schedules. They were eager to find solutions that not only fit the parameters but also satisfied their individual interests. Teachers enjoyed watching students actively engage with the activity, share ideas with neighboring students, ask questions, and ultimately reach a solution that they were excited to communicate.

What *keeps* it worthwhile?

Even such an engaging activity requires diligence on the teacher's part to keep the task at a high cognitive level. Features of the implementation—such as scaffolding, honoring a variety of solutions, and pressing students for explanation—are fundamental to maintaining a high learning value because they keep students, rather than the teacher, doing the reasoning. To support the students' development of mathematical problem-solving skills, teachers must prevent themselves from taking over the thinking (Herbel-Eisenmann and Breyfogle 2005).

1. Scaffold student thinking. During the implementation of this activity with a fourth-grade class, scaffolding proved to be a successful tool in promoting student reasoning through the elapsed-time challenge. The following exchange occurred as a student calculated the elapsed time from 11:10 to 12:02:

John: I can't figure out how much time I spent on math.
Teacher: Well, what is one hour from 11:10?
John: 12:10 … [*pausing*] so, it's less than one hour.
Teacher: Yes, now we need to find out how much less than one hour.
John [*pausing*]*:* Eight minutes.
Teacher: Ok, then how much time has elapsed from 11:10 to 12:02?
John: Sixty minutes minus eight is [*pausing*] fifty-two! Fifty-two minutes!

By suggesting that the student think about a one-hour chunk as a benchmark, Williams allowed John to do some of his own thinking. She also allowed think time for him to answer the questions she posed. When John answered, "12:10," she did not immediately jump in to ask another question. She provided a series of scaffold questions or statements that did not funnel his responses; he could use them as models in the future.

2. Honor a variety of strategies. Allowing for a variety of strategies is another way to maintain the level of the task. Students used many strategies to determine their schedules. For example, in contrast to John's approach of thinking about adding an hour and then subtracting to get to a benchmark number, Amanda used an additive approach. She calculated the duration of recess, from 12:44 to 1:10, by first counting on one minute from 12:44 to 12:45 (which can be considered a benchmark number or time), and then counting on using five-minute intervals until she reached 1:10. Amanda also used the addition problem $5 + 42 = 47$ to calculate the elapsed time from 1:55 to 2:42. In this calculation, she arrived at her answer of forty-seven minutes by considering what it would take to get from 1:55 to 2:00 (five minutes) and then adding the number of minutes to 2:42.

3. Press for explanations. Continuous teacher questioning and pressing for explanation and justification is another aspect of effective implementation of a worthwhile task (Stein et al. 2000). This technique keeps the emphasis on solution strategies and reasoning (rather than the correct answer) and plays an important role in supporting mathematical reasoning (NCTM 1991). During the lesson, one student correctly calculated the elapsed time from 1:54 to 2:42 by providing the answer of forty-eight minutes to the class. Rather than acknowledging the correct answer and progressing with the lesson, Williams asked the student to explain his solution process:

Teacher: How did you find that answer?
Tom: Well, one hour from 1:54 is 2:54. Then I did fifty-four minus forty-two and got twelve.
Teacher: Why did you do fifty-four minus forty-two?
Tom: To figure out how much less than an hour it was. It was twelve minutes less than an hour. Then I did sixty minus twelve and got forty-eight, so the answer is forty-eight minutes.

A brief conversation requiring students to explain their thought processes can provide valuable insights to all. The process of communicating mathematically will benefit the student with the opportunity to clarify his thinking. The teacher benefits because she gains insight into this student's thinking and thought processes, which could guide further instruction. The class members gain from these exchanges because they are able to hear other methods that possibly differ from their own thinking.

Reflecting on the Implementation

The reflection that occurs after the lesson is also critical in making improvements so that teachers can become more effective at keeping the tasks' cognitive demands at a high level. In this case, as Breyfogle and Williams reflected collaboratively on the implementation of the elapsed-time tasks, they came to several observations. First, in the exchange with John described previously, it might have been possible to gain an even greater insight into the child's mind and provide a richer learning experience. If Williams had instead asked questions such as, "What are you thinking?" or "Do you have an idea of how you could proceed?" she could have come to understand what John already knows in order to guide, rather than lead, him to developing his own way to solve the problem. Additionally, in the second exchange, Williams might have pressed Tom's thinking more by asking exactly what he meant when he said, "Fifty-four minus forty-two," because many of the other students in class may have been thinking, "Fifty-four take away what will get me to forty-two?" Having Tom clarify his interpretation of "fifty-four minus forty-two" as the difference between forty-two and fifty-four would have been helpful to the other students. Continuing this conversation to involve some discussion of the different ways you can interpret subtraction, accompanied by appropriate representations, could have helped clarify Tom's thinking and perhaps provided more opportunity for other students to understand it.

Conclusion

The point of creating this worthwhile task was for students to develop ways to accurately and flexibly determine elapsed time. Student work showed that each was able to successfully complete the task. For example, at the beginning of class, Tony quickly responded, "Four o'clock," when students were asked to calculate the elapsed time of a party beginning at 1:30 and ending three hours later. At the end of class, Tony was able to successfully complete this task. The task challenged the entire class to make connections to prior knowledge, think critically, and communicate mathematically. It not only encouraged them to make sense of elapsed time but also resulted in a noticeably positive change in student engagement and learning. By incorporating the key elements of a worthwhile task and effective implementation and reflection techniques, the elapsed-time task created a desirable learning environment that is an essential foundation for understanding mathematics.

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A Tale of Two Tasks: K-3

Grades K-2

Task A
If I have two pennies, a nickel, two dimes, and a quarter, how money do I have?

Source: Hull, Miles, & Balka, 2014, pg. 23
2.MD.C.8

Task B
I have 5 coins in my pocket. The coins may only be pennies, nickels, dimes, or quarters. If I reach into my pocket and pull out three coins, how much money might I have in my hand?

Source: Hull, Miles, & Balka, 2014, pg. 23
2.MD.C.8

Grades 2-3

Task A
Find the difference 731 – 256 =

Source: *Making Sense of Mathematics: Reasoning and Discourse*, Scholastic, 2012, pg. 10
2.NBT.B.7 /3.NBT.A.2

Task B
Arrange the digits so the difference is between 100 and 200.
731- 256 =

Source: *Making Sense of Mathematics: Reasoning and Discourse*, Scholastic, 2012, pg. 10
2.NBT.B.7 /3.NBT.A.2

A Tale of Two Tasks: Grades 4-5

Grades 4-5

Task A

Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

Source: Stein, Smith, Henningsen, & Silver, 2000, pg. 1
4.MD.A.3

Task B

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits.

a.) If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?
b.) How long would each of the sides of the pen be if they had only 16 feet of fencing?
c.) How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

Source: Stein, Smith, Henningsen, & Silver, 2000, pg. 1
4.MD.A.3

A Tale of Two Tasks: Grades 6-7

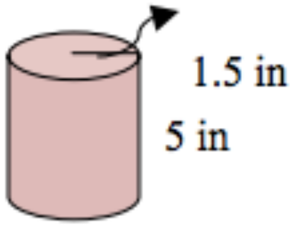
Grades 6-7

Task A The Smith family are aged 3, 8, 9, 10, and 15. What is their average age? Source: *Making Sense of Mathematics: Reasoning and Discourse*, Scholastic, 2012, pg. 10 *6.SP.A.3*
Task B Four people in this room have an average height of 148 cm. You are one of those. Find the other three. Explain your approach. Referenced in Sullivan, P, Clarke, D., Clarke, B., (2013.) *Teaching with Tasks for Effective Mathematics Learning.* New York: Springer *6.SP.A.3*

A Tale of Two Tasks: Grade 8

Grade 8

Task A



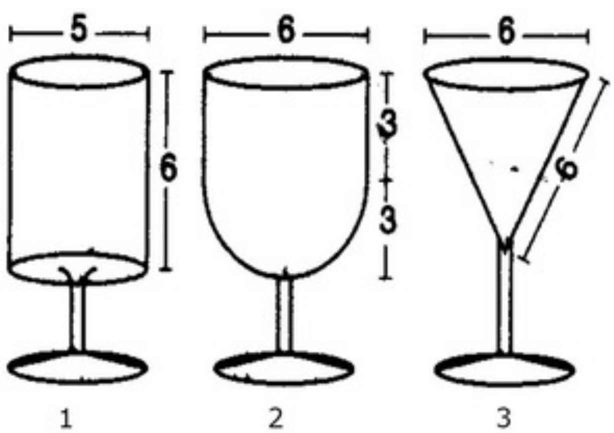
A) What is the volume of the smallest box Sam could build to hold the candle? Show the mathematics you used to determine your answer.

B) What is the surface area for this box? Show the mathematics you used to determine your answer

Source: NC Department of Public Instruction
8.G.C.9

Task B

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.



a.) What is the height of Glass 3?
b.) What is the volume of each?
c.) If Glass 1 has volume *V*, express volume of Glasses 2 and 3 in terms of *V*.
d.) When Glass 1 is $\frac{1}{2}$ full, the height of the liquid is 3 cm. What are the heights of the liquid in Glasses 2 and 3 when they are $\frac{1}{2}$ full?

Adapted from Illustrative Mathematics
8.G.C.9

A Tale of Two Tasks: High School

High School

Task A

Solve each of the following systems:

$$-4x - 2y = -12$$
$$4x + 8y = -24$$

$$x - y = 11$$
$$2x + y = 19$$

$$8x + y = -1$$
$$-3x + y = -5$$

$$5x + y = 9$$
$$10x - 7y = -18$$

Source: Leinwand, S., Brahier, D., and Huinker, D., Principles to Action, pg. 20
A.REI.C.6

Task B

You are trying to decide which two smartphone plans would be better. Plan A charges a basic fee of $30 per month and 10 cents per text message. Plan B charges a basic fee of $50 per month and 5 cents per text message.

a) How many text messages would you need to send per month for Plan B to be the better option? Explain your decision?
b) If the cell phone company decided to offer unlimited texts for $80 per month, do you think that you would change your smartphone plan? Use mathematical reasoning to support your decision.

Adapted from Illustrative Mathematics and *Leinwand, S., Brahier, D., and Huinker, D., Principles to Action, 2014, pg. 20*
A.REI.C.6

Levels of Task Cognitive Demand

Different levels of tasks lead to different opportunities for student learning. Higher-level demand tasks connect procedures to important mathematical concepts and representations to develop those concepts. These tasks provide opportunities for students to reason mathematically and make sense of mathematics.

Low-Level Cognitive Demand	High-Level Cognitive Demand
If I have two pennies, a nickel, two dimes, and a quarter, how money do I have? *Source: Hull, Miles, & Balka, 2014, pg. 23*	I have 5 coins in my pocket. The coins may only be pennies, nickels, dimes, or quarters. If I reach into my pocket and pull out three coins, how much money might I have in my hand? *Source: Hull, Miles, & Balka, 2014, pg. 23*
Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase? *Source: Stein, Smith, Henningsen, & Silver, 2000, pg. 1*	Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits. a.) If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be? b.) How long would each of the sides of the pen be if they had only 16 feet of fencing? c.) How would go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it. *Source: Stein, Smith, Henningsen, & Silver, 2000, pg. 2*

Strategies for Transforming Tasks

Have a low level task? Try these strategies out for transforming it into a more worthwhile task.

Turn Around the Question

Process	Example
Step 1: Identify a topic.	The topic for tomorrow is averages.
Step 2: Think of a low-level question and write down the answer.	A low level question might be: The children in the Smith family are aged 3, 8, 9, 10, and 15. What is their average age?
Step 3: Make up a question that includes or addresses the answer to the original question.	A high-level question could be: There are five children in a family. Their average age is 9. How old might the children be?

Sullivan, P, & Lilburn, P., *Good Questions for Math Teaching: Why Ask Them and What to Ask: Grades K-6*, (2002.)

Adapting a Standard Problem

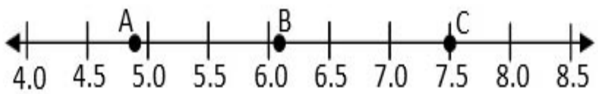
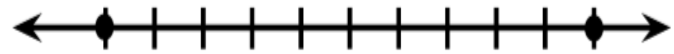
Process	Example
Step 1: Identify a topic.	The topic for my lesson is multiplication.
Step 2: Think of a standard problem and write down the answer.	A standard problem might be: Rod has 4 packages of pencils. There are 6 pencils in each package. How many pencils does Rod have in all?
Step 3: Adapt it to make it an open problem with multiple approaches and/or solutions.	An open problem could be: Rod has some packages of pencils. There are 2 more pencils in each package than the number of packages. How many pencils does Rod have in all?

Small, Marian, *Good Questions: Great Ways to Differentiate Mathematics Instruction*, (2012.)

Add a comparison component (e.g., Which is a better deal? Why? Which is correct? Why?)

Original Task	Transformed Task
The jSongs online music library is running a promotion: this week only, it costs $7.00 to sign up for the service, but you can download as many apps as you like for $1.50 each. Write an equation and draw a graph to represent how much someone spends to join jSongs and download their apps this week.	The jSongs and K-Songs are online music libraries. Both companies are running a promotion this week. jSongs $7 sign-up fee + $1.50 per app K-Songs $10 sign-up fee + $0.85 per app Which plan would you recommend to a friend? Write a paragraph explaining your recommendation. Use the slope and y-intercept in explaining your recommendation.

Add, "Explain why your answer is reasonable."

Original Task	Transformed Task
Match each irrational number with its correct point on the number line. $\sqrt{56}, \quad \sqrt{24}, \quad \sqrt{37}$  	Estimate the value of each of the following numbers and place them on the number line. Explain in writing why each of your number placements is reasonable. $\sqrt{37}, \ \sqrt{24}, \quad \sqrt{42}, \ \sqrt{2} + 7, \ 2\pi$ 

Add or remove constraints and conditions

Original Task	Transformed Task
Your class is taking a 200-mile bus trip to Washington, D.C. When the bust stops for a lunch break, you have traveled 160 miles. What percent of the trip have you traveled?	For school field trips between 150 and 250 miles away, the school requires the following: A stop to check the bus's tire pressure 20% of the way. A restroom stop 60% of the way. A food stop 80% of the way. At what number of miles should the bus driver plan to stop for a trip of 150 miles, 200 miles, and 250 miles?

Open up the task

<table>
<tr><th>Original Task</th><th>Transformed Task</th></tr>
<tr>
<td>James earns $90 in 5 hours mowing lawns. At this rate, how much will he earn in 8 hours?</td>
<td>a.) James earns $84 in 8 hours mowing lawns. Use the table to show how much he would earn for each given number of hours.
<table>
<tr><td>Hours Worked</td><td>10</td><td>8.5</td><td>7.5</td><td>6</td><td>3.5</td><td>2.5</td></tr>
<tr><td>Money Earned</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
b.) James wants to make more money in fewer hours. Suggest a new rate and use the table to show how much James would earn for each given number of hours at the new rate.
<table>
<tr><td>Hours Worked</td><td>3</td><td>5</td><td>2.5</td><td>4</td><td>7.5</td><td>8</td></tr>
<tr><td>Money Earned</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
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Notes

Notes

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