Connecticut Common Core Algebra 1 Curriculum

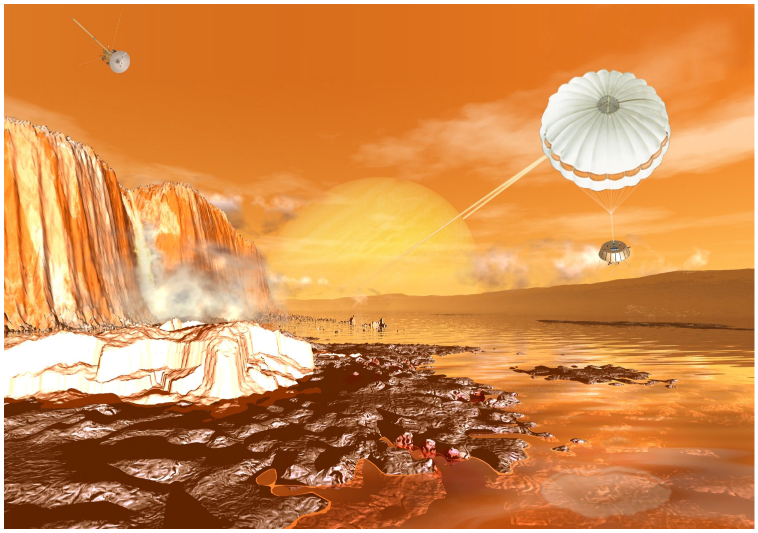
Professional Development Materials

Unit 1 Patterns

Contents

*** These items are found only on the password-protected web site.**

Hydrocarbons on Titan



Artist's Rendition of Hyugens Probe on the Surface of Titan

Hydrocarbons are molecules made up only of hydrogen and carbon. They are found on Earth in crude oil and natural gas, and are the primary source of energy used throughout the world. Due to an increasing demand for energy, the Earth's supply of hydrocarbon fuel is diminishing.

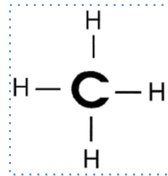
In 2004, the Cassini spacecraft orbited Titan, Saturn's largest moon, sending images of Titan's atmosphere and surface to Earth. The images confirmed that Titan has lakes which contain hydrocarbons. These hydrocarbons are gases on Earth, but due to Titan's surface temperature, they are liquids, or even solids, on Titan. The image above is an artist's rendition of the surface of Titan with the Hyugens probe parachuting toward Titan's surface.

You have been selected for a mission to Titan departing Earth in 2020. The goal of the mission is to explore and catalog the mixture of simple hydrocarbons in the lakes of Titan and report your findings. Simple hydrocarbons are made up of specific patterns of hydrogen and carbon atoms. Each simple hydrocarbon has a name and its own properties.

As a first step, your task is to build models of hydrocarbons and find their mathematical pattern. You must use the following guidelines:

- Carbon atoms are represented by black Styrofoam spheres.
- Hydrogen atoms are represented by yellow Styrofoam spheres.
- Each carbon atom must have four links. Each link can connect another carbon atom or a hydrogen atom. The carbon atoms must lie along a straight line. This is called a straight-chain hydrocarbon.
- Each hydrogen atom can only have one link. Hydrogen atoms must be connected to a carbon atom.

The first hydrocarbon is called methane and it looks like this:



1. Build a model of methane using one black Styrofoam (carbon atom) and four yellow Styrofoam (hydrogen atoms). Check with your mission instructor (teacher) before continuing.

2. Build a model with 2 carbon atoms (black). Continue to follow the guidelines provided on the previous page. Check with your mission instructor (teacher) before continuing. Then count the number of hydrogen atoms.

3. Continue building models using three and four carbon atoms (black). Follow the guidelines for hydrocarbons listed on the previous page.

4. Complete the following table:

Hydrocarbon Molecules		
Name of Molecule	**Number of Carbon Atoms**	**Number of Hydrogen Atoms**
Methane	1	4
Ethane	2	
Propane	3	
Butane	4	
Pentane		

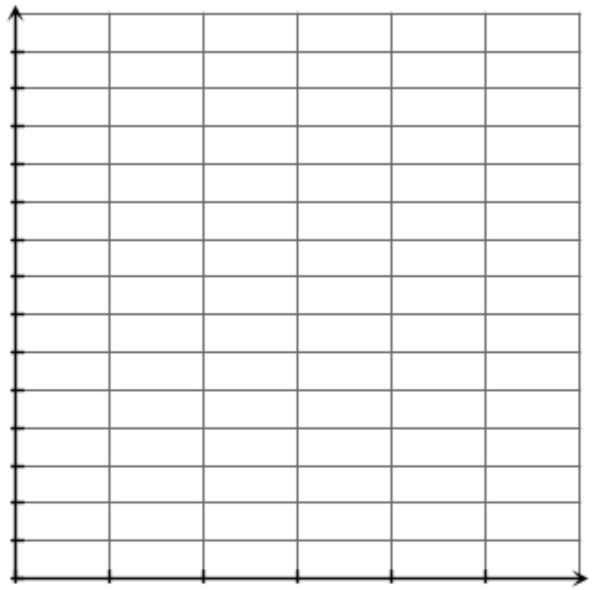
5. Draw a rough stick model of a hydrocarbon with five carbon atoms using *C* for carbon atoms and *H* for hydrogen atoms.

6. Complete the following sentence:

   As the number of carbon atoms increases by 1, the number of hydrogen atoms

   ___________________________________.

7. Use the coordinate plane below to graph the relationship between the number of carbon atoms and the number of hydrogen atoms in the hydrocarbons. Label and scale the axes appropriately.



8. Are you able to extend the graph to determine how many hydrogen atoms would be in a hydrocarbon with six carbon atoms? If so, what would you find?

9. What would happen if you extended the graph to the left? Is the result meaningful?

10. If we know there are 20 hydrogen atoms in a hydrocarbon with 9 carbon atoms, how many hydrogen atoms are there in a hydrocarbon with 10 carbon atoms?

11. Which of the following equations can be used to find the number of hydrogen atoms if we know the number of carbon atoms? Use the table to verify your answer.

(A) $h = 4c + 0$ (B) $h = 2c + 4$ (C) $h = 2c + 2$

12. What do the variables h and c represent in the previous equations?

13. Use your equation from Question 11 to find the number of hydrogen atoms in a hydrocarbon with 20 carbon atoms.

14. Suppose there are 22 hydrogen atoms in a hydrocarbon. Use your equation from Question 11 to determine how many carbon atoms would be present.

15. Is it possible to have a hydrocarbon molecule with 25 hydrogen atoms? Explain.

In this activity, we examined many different representations of the relationship between the number of carbon atoms and hydrogen atoms. We used a table (Question 4), verbal description (Question 6), graph (Question 7), and an equation (Question 11).

16. Which representation did you find most useful? Why?

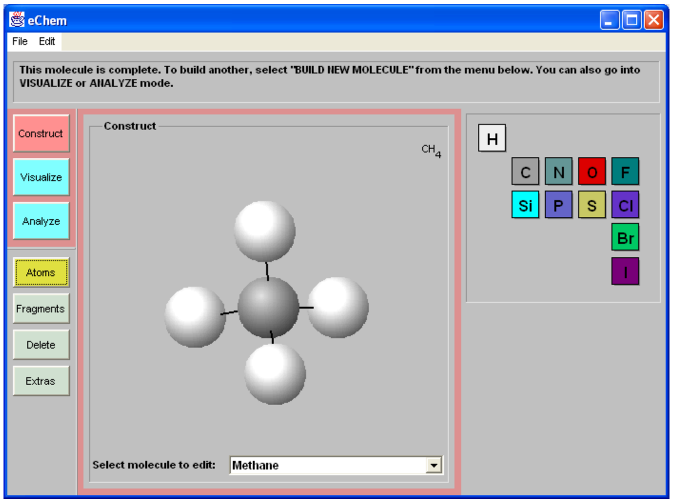
17. Which representation is the most difficult to understand? Why?

18. What similarities and differences do you see in the four representations?

Using eChem to Model Molecules

eChem is a free online software that allows students to create virtual, three-dimensional models of molecules. eChem was created by the Center for Highly Interactive Classrooms, Curricula & Computing in Education (Hi-Ce) at the University of Michigan. It is available at:

- eChem Structure Building Applet: http://www.sciencegeek.net/eChem/eChem.html
- hi-ce.org: http://hi-ce.org/echem/index.html



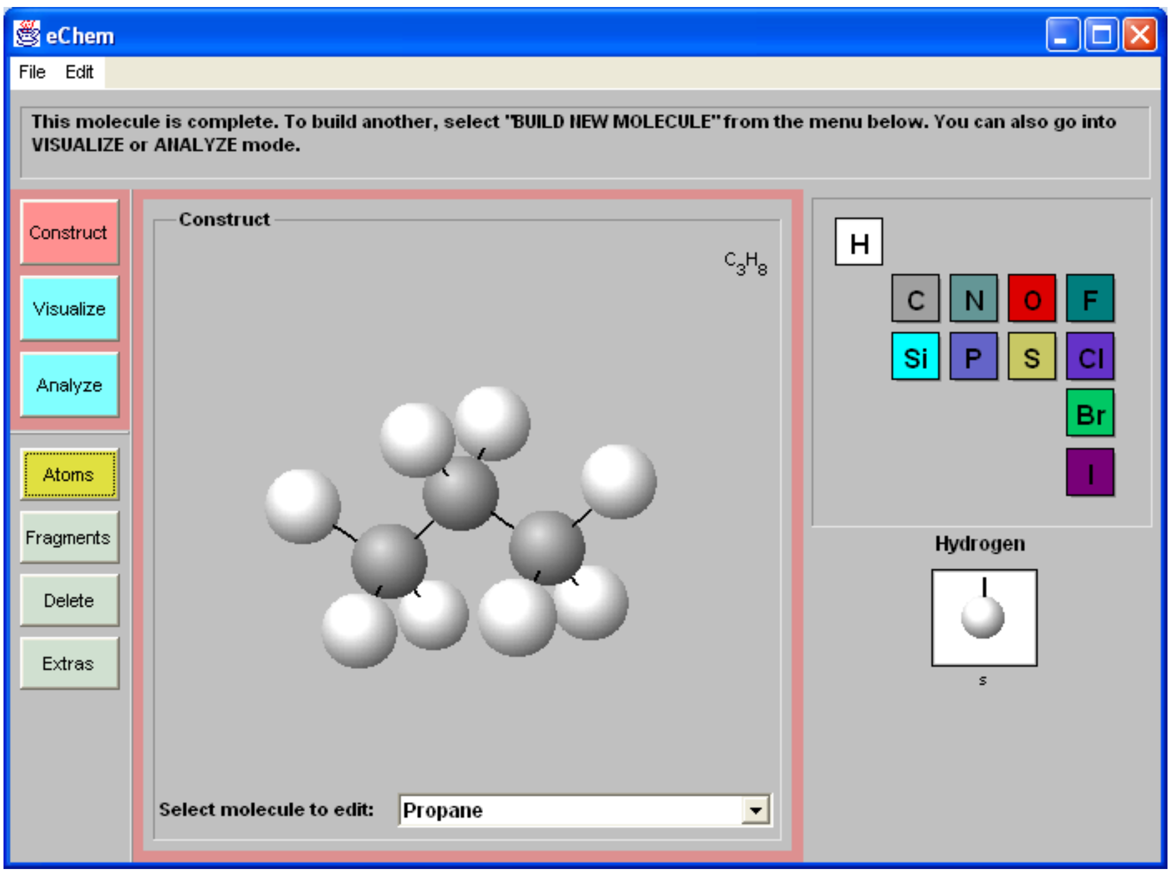
<u>eChem Instructions:</u>

1. Run eChem from one of the websites listed above (or from the computer if installed).
2. To construct the model of methane (CH_4), start on the left side of the screen. When you see “Construct,” “Visualize,” and “Analyze.”
3. Click “Construct.”
4. In the popup window, enter the name of molecule: methane.
5. Click “Atoms,” and select “C” (carbon) from element table on the right side of the screen.
6. Select bond arrangement. In this case, select “tetrahedral” as the hybridization of carbon and click on the canvas (the large area in the middle of the window).
7. Select “H” (hydrogen) from the atom table.
8. Add hydrogen atoms to the carbon’s bond by clicking on the bonds of carbon.

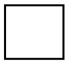
Notes:

1. You can see a rotating structure; just hold down the mouse button and move the cursor.
2. Did you attach a wrong atom on carbon (like adding an oxygen atom for methane)? Select "Delete" on the left side of screen, and click the atom ball you want to erase.
3. If you are going construct a new model, pull down the bottom menu on the canvas, and select "build a new molecule."
4. The "Visualize" button allows you to view the molecule in three different ways.
5. If you entered multiple molecules, the "Analyze" button allows you to check the pattern by choosing "Carbon Count" and "Atom Count" as column headers.

Enjoy!

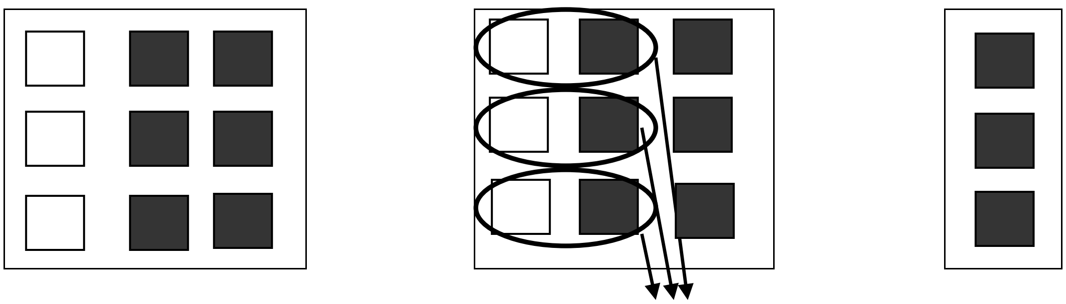


Algebra Tiles and Integers

Key	Rules to Remember
 = 1	The sum and difference of two numbers can be positive, negative, or zero.
	The sum of an integer and its opposite is zero.
 = -1	Subtracting an integer is the same as adding the opposite of the number.

You can use algebra tiles to help you **add integers**.

Here is how to model the solution to the problem 3 + (-6)



3 + (-6) Remove pairs that equal zero -3

1. Use algebra tiles to model and find each sum.

(A) 4 + (-2) (B) 1 + (-4) (C) -2 + 2

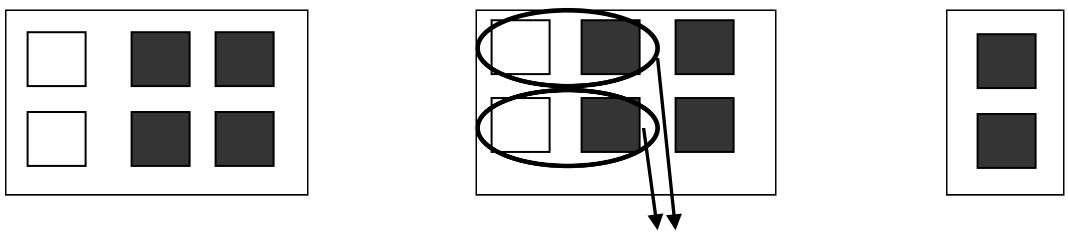
(D) -1 + 4 (E) -5 + (-5) (F) -3 + -2

You can use algebra tiles to **subtract integers**.

Here is how to model a solution to the problem 2 – 4.

First change the subtraction problem to an addition problem by keeping the sign of the first integer, switching the operation to addition then changing the sign of the second integer.

$$2 - 4 = 2 + (-4)$$



2 + (-4) Remove pairs that equal zero -2

2. Use algebra tiles to model and find each difference. First change the subtraction to an addition. Remember the rule: $-(-n) = n$. This means that the opposite of the opposite of a number is the original number,

(A) 2 – (-4) (B) -2 – 1 (C) -1 – 4

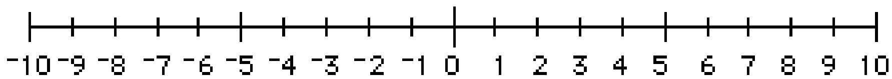
(D) -4 – 2 (E) -3 – (-5) (F) -3 – 2

Patterns in Signed Numbers

Symbols for positive and negative numbers:

Positive numbers may be indicated with a raised "plus" sign, for example, $^{+}3$.
Negative numbers may be indicated with a raised "minus" sign, for example, $^{-}4$.
We often write a positive number without the raised "plus" sign, for example, $3 = {}^{+}3$.

Positive and negative integers may be shown on a number line.



Addition of Integers	Key words: add, combine, plus Symbol for addition: a + b means "a plus b"

You may already be familiar with the rules for combining two signed numbers. You can also discover these rules by observing patterns.

(1) Complete this pattern

3 + 5 = ________

3 + 4 = ________

3 + 3 = ________

3 + 2 = ________

3 + 1 = ________

3 + 0 = ________

3 + -1 = ________

3 + -2 = ________

3 + -3 = ________

3 + -4 = ________

3 + -5 = ________

(2) Complete this pattern

-2 + 5 = ________

-2 + 4 = ________

-2 + 3 = ________

-2 + 2 = ________

-2 + 1 = ________

-2 + 0 = ________

-2 + -1 = ________

-2 + -2 = ________

-2 + -3 = ________

-2 + -4 = ________

-2 + -5 = ________

Now state the rules for adding (combining) two signed numbers:

If both numbers are positive, ____

If both numbers are negative, ____

If one number is positive and the other is negative, ____________________________________

Subtraction of Integers	Key words: Subtract, find the difference, minus Symbol for subtraction: a – b means "a minus b"

Discover the rules for subtraction by observing patterns.

(1) Complete this subtraction pattern

9 – 4 = ________

9 – 3 = ________

9 – 2 = ________

9 – 1 = ________

9 – 0 = ________

9 – -1 = ________

9 – -2 = ________

9 – -3 = ________

9 – -4 = ________

(2) Complete this addition pattern

9 + -4= ________

9 + -3= ________

9 + -2 = ________

9 + -1 = ________

9 + 0 = ________

9 + 1 = ________

9 + 2 = ________

9 + 3 = ________

9 + 4 = ________

(3) Complete this subtraction pattern

-2 – 4 = ________ (think "4 less than -2)

-2 – 3 = ________

-2 – 2 = ________

-2 – 1 = ________

-2 – 0 = ________

-2 – -1 = ________

-2 – -2 = ________

-2 – -3 = ________

-2 – -4 = ________

(4) Complete this addition pattern

-2 + -4= ________

-2 + -3= ________

-2 + -2 = ________

-2 + -1 = ________

-2 + 0 = ________

-2 + 1 = ________

-2 + 2 = ________

-2 + 3 = ________

-2 + 4 = ________

Compare answers in pattern 1 and in pattern 2. ____________________

Compare answers in pattern 3 and in pattern 4. ____________________

How are subtraction and addition related? ____________________

Describe a rule for subtracting signed numbers: ____________________

____

Multiplication of Integers	Key words: multiply, find the product, times Multiplication symbols: x, *, ·

Discover the rules by observing patterns.

5 * 4 = ________	-5 * 4= ________
5 * 3 = ________	-5 * 3 = ________
5 * 2 = ________	-5 * 2 = ________
5 * 1 = ________	-5 * 1 = ________
5 * 0 = ________	-5 * 0 = ________
5 * -1 = ________	-5 * -1 = ________
5 * -2 = ________	-5 * -2 = ________
5 * -3 = ________	-5 * -3 = ________

Division of Integers	Key words: divide by, find the quotient Multiplication symbols: ÷, /

Discover the rules by observing patterns.

20 ÷ 5 = ________	20 ÷ -5 = ________
15 ÷ 5= ________	15 ÷ -5 = ________
10 ÷ 5 = ________	10 ÷ -5 = ________
5 ÷ 5 = ________	5 ÷ -5 = ________
0 ÷ 5 = ________	0 ÷ -5 = ________
-5 ÷ 5 = ________	-5 ÷ -5 = ________

-10 ÷ 5 = ________ -10 ÷ -5 = ________

Now state the rules for multiplying and dividing two signed numbers:

If both numbers are positive, ____

If both numbers are negative, ____

If one number is positive and the other is negative, ____________________________________

Summarize what you found in the table below.

	Both Positive	**Both Negative**	**One Positive, One Negative**
Addition			
Subtraction			
Multiplication			
Division			

Bingo with Order of Operations

Evaluate the following expressions using the Order of Operations.

1) $12 \div 3 \times 4$

2) $4 + 10 - 5 + 8$

3) $2 + 9 \div 3 - 5 + 6 \times 5 \div 2$

Directions:
The object of the game is to get five numbers in a row, vertically, horizontally, or diagonally. Each student will get their own Bingo card.

- Choose one space on your Bingo card as the "free" space and write the word FREE.
- Choose numbers to write into the other 24 boxes on your Bingo card. Make sure you choose numbers within the ranges given at the top of each column.
- You are not allowed to repeat any numbers.

Example of the Bingo card:

B (1 – 10)	**I** (11 – 20)	**N** (21 – 30)	**G** (31 – 40)	**O** (41 – 50)

B (1 – 10)	**I** (11 – 20)	**N** (21 – 30)	**G** (31 – 40)	**O** (41 – 50)

B (1 – 10)	**I** (11 – 20)	**N** (21 – 30)	**G** (31 – 40)	**O** (41 – 50)

Expression	Answer
(3 × 3) ÷ (3 × 3)	1
5 ÷ 5 + 5 ÷ 5	2
3 + 4 – 4 × 1	3
(6 + 25 – 7) ÷ 6	4
8 – 1 – (18 – 2) ÷ 8	5
(9 × 2) ÷ (2 + 1)	6
2 × 7 ÷ 2	7
9 – (9 – (9 – 1))	8
(30 – 3) ÷ 3	9
9 + 6 ÷ (8 – 2)	10
6 – 1 + 6	11
5 × 4 – 8	12
4 + 4 + 9 – 4	13

Expression	Answer
1 × 6 + 4 × 5	26
4 + 2 × 15 – 7	27
2 × 2 × (3 + 4)	28
9 + 10 + 10	29
2 × 5 + 4 × 5	30
3 × 3 × 3 + 4	31
12 × 3 – 2 × 2	32
7 – 2 + 4 × 7	33
10 × 9 – 8 × 7	34
(4 – 3)((3 + 5) – 1) × 5	35
4(9 ÷ 3 × 9 ÷ 3)	36
2 + 7 × 5	37
(5 × 4 × 4 – 4) ÷ 2	38

$(6 - 4) \times 49 \div 7$	14		$3(6 + 7)$	39
$72 \div 9 + 7$	15		$(4 + 3 + 2 + 1) \times 4$	40
$2 \times 2 \times 2 \times 2$	16		$7 \times 9 - 7 - 3 \times 5$	41
$15 + 40 \div 20$	17		$49 \div 7 \times 60 \div (2 \times 5)$	42
$2 \div 2 + 7 + 4 + 4 + 2$	18		$7 \times 7 - (8 - 2)$	43
$9 + 9 + 6 - 5$	19		$4 \times (3 \times 2 \times 2 - 1)$	44
$(4 - 1 + 8 \div 8) \times 5$	20		$5(10 - 1)$	45
$20 + 16 - 15$	21		$3 \times 3 \times 3 + 10 + 9$	46
$6 \times 7 - 5 \times 4$	22		$5 \times (8 \times 5) \div 4 - (7 - 4)$	47
$5 - 15 \div 5 + 7 \times 3$	23		$-3 \times 2 \times 2(-3 - 1)$	48
$4(4 \div 2 + 4)$	24		$7 \times 7 \times 4 \div 4$	49
$(1 + 4) \times (4 + 1)$	25		$1 + 7 \times 7$	50

Recursive and Explicit Rules for Arithmetic Sequences

A **sequence** is a list of numbers which follow a specific pattern. Each number in the sequence is called a term. An **arithmetic sequence** is a sequence in which consecutive terms differ by a constant amount.

1. Find the next three terms of each sequence.

   (a) 2, 5, 8, 11, ______, ______, ______ (b) 12, 7, 2, -3, ______, ______, ______

A **recursive rule** for a sequence is a rule which uses the value of one term (or the value of multiple terms) in the sequence to define the value of the next term in the sequence. You must state a beginning value.

An **explicit rule** for a sequence is a formula that determines any term in the sequence. Depending on your data, the beginning term could be the 0^{th} or 1^{st} term.

2. Every week, Jane, a travel agent, gets paid \$900 (her base salary) plus an additional \$100 for each cruise she books.

   (a) Complete the table below by identifying her salary based on the number of cruises she books in a week.

Cruises	Salary	Recursive Pattern
0	900	900
1	1000	900 + 100
2		1000 + 100
3		
4		

   (b) What is a recursive rule for the sequence of salaries?

   (c) Write an explicit rule for the sequence of salaries. Let *c* represent the number of cruises she books and *s* represent her salary.

   (d) Find Jane's salary when she books 8 cruises.

3. You bring $20 to a carnival to buy tickets for an arcade game. You spend $1.50 for each ticket. You play the game several times until you win.

(a) Complete the table below by identifying the amount of money you have left after buying tickets for different numbers of games.

Games	Amount of Money	Recursive Pattern
0	20.00	20.00
1	18.50	20.00 – 1.50
2		
3		
4		

(b) What is a recursive rule for the sequence of amounts?

(c) Write an explicit rule for the sequence of amounts. Let *a* represent the amount of money you have left and *g* represent the number of games.

(d) How much money do you have left after 8 games?

4. You buy an Xbox 360 game system for $250 and you spend $50 for each additional game.

(a) Complete the table below by identifying the total cost for the Xbox 360 and the indicated number of games.

Games	Total Cost	Recursive Pattern
0	250	250
1	300	250 + 50
2		
3		
4		

(b) What is a recursive rule for the sequence of total costs?

(c) Write an explicit rule for the sequence of total costs. Let *t* represent the total costs and *g* represent the number of video games purchased.

(d) What is the total cost if you buy 10 games?

5. Identify a recursive rule and an explicit rule for the sequence: 2, 5, 8, 11, …

6. Identify a recursive rule and an explicit rule for the sequence: 12, 7, 2, -3,…

Doubling Your Money

A **geometric sequence** is a sequence in which there is a common ratio among each pair of consecutive terms.

Congratulations! You just won the $500 first prize in a poetry writing contest. If you take the $500 you won and invest it in a mutual fund earning 8% interest per year, how long will it take for your money to double?

1. Explore different ways to solve this problem by creating a table.

Year	Value of Investment
0	$500
1	
2	
3	
4	
5	
6	
7	
8	
9	

2. What is the recursive rule for this geometric sequence?

3. After 15 years, your $500 investment will be worth $1586.08. What will your investment be worth after 16 years?

4. Graph your table of values on the coordinate plane below. Label and scale the axes appropriately.