Connecticut Common Core Algebra 1 Curriculum

Professional Development Materials

Unit 3 Functions

Contents

*** These items appear only on the password-protected web site.**

Is it a Function?

Functions

In order for a relation to be a function, each ___________________________ must be mapped to one ____________________.

1. Identify whether or not each relation is a function.

(a)

x	–2	–1	0	1	2
y	0	5	6	0	3

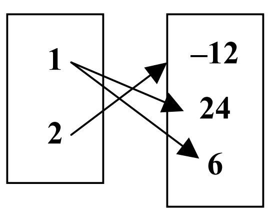
(b)

x	–2	–1	0	1	–2
y	4	–1	3	2	1

(c)

State	Maine	New Hampshire	Vermont	Massachusetts	Rhode Island	Connecticut
Capital	Augusta	Concord	Montpelier	Boston	Providence	Hartford

(d)



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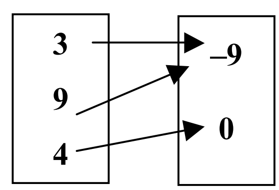

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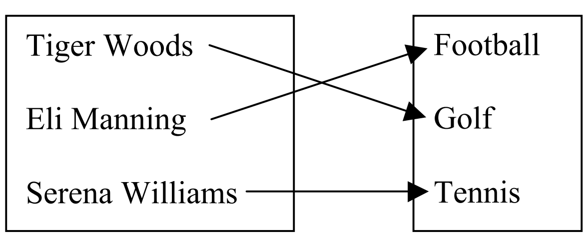


(g) {(2, 3), (4, 3), (-1, 0), (6, 1), (-2, 8)}

(h) {(3, 4), (5, -2), (7, -1), (3, 3), (1, 5)}

Domain and Range

The domain of a function is ____

The range of a function is ____

2. For each function in Exercise 1, identify the domain and range.

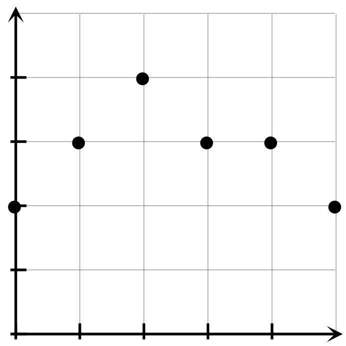
Letter of the function	Domain	Range
a		
b		
c		
d		
e		
f		
g		
h		

3. Create your own function below and explain why it is a function.

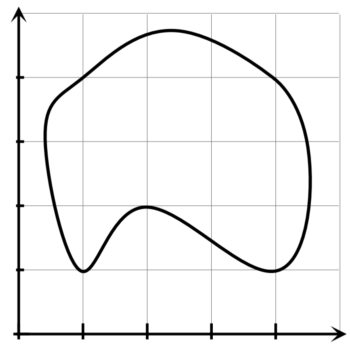
4. Give an example of a relation that is not a function and explain why not.

5. Identify whether or not each graph represents a function.

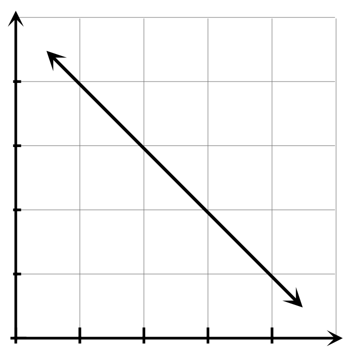
(a)



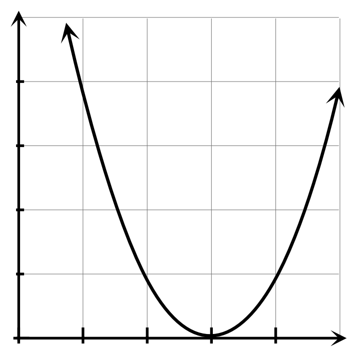
(b)



(c)



(d)



6. For each of the graphs in Exercise 5, find three ordered pairs and display them in a table.

(a)

x	*y*

(b)

x	*y*

(c)

x	*y*

(d)

x	*y*

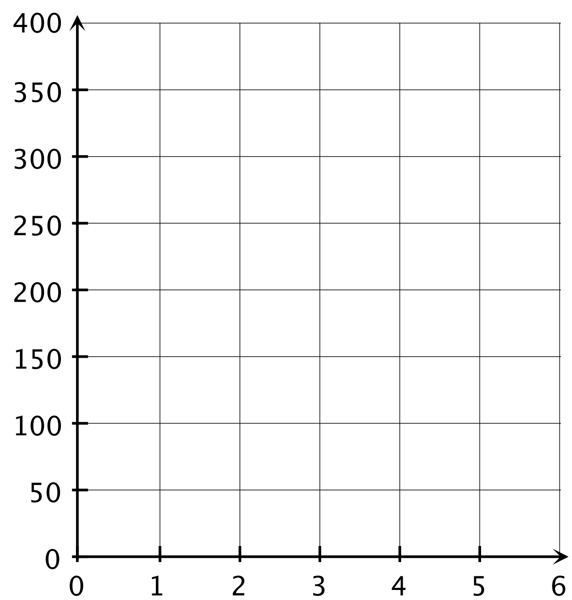
7. You are at an altitude of 250 feet in a hot-air balloon. You turn the burner on high and rise at a rate of 20 feet per minute for 5 minutes. Your altitude h after you have risen for t minutes is given by the function $h = 250 + 20t$.

   (a) Make a table to show the altitude as a function of the number of minutes you have traveled.

   (b) Graph your data points—make sure to label the units on your axes.

   (c) Does it make sense to connect the points on the graph? Explain.

# of minutes	**Altitude (in feet)**
0	
1	
1.5	
2	
3	
4	
5	



   (d) After 5 minutes, you turn the burner to low. This gives just enough heat to keep the balloon from falling but not enough to make it rise any higher. Plot a point on the graph to show how high the balloon is after 6 minutes.

   (e) Does it make sense to connect the point plotted in part (d) to the rest of the graph? Explain.

   (f) What is the domain of the function?

   (g) What is the range of the function?

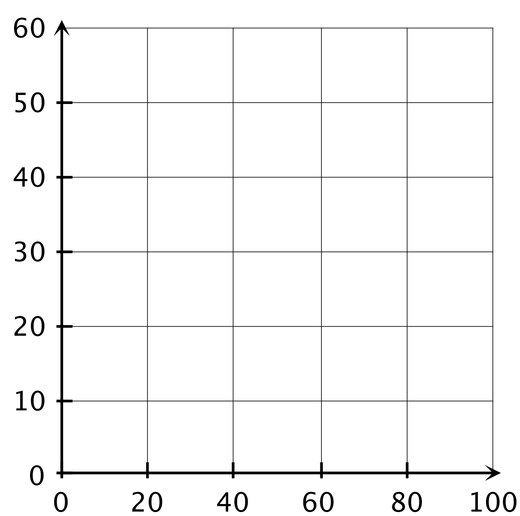
8. As a scuba diver dives deeper and deeper into the ocean, the pressure of the water on his body steadily increases. The pressure at the surface of the water is 14.7 pounds per square inch (psi). The pressure increases at a rate of 0.445 psi for each foot you descend.

(a) Write an equation that represents the pressure P as a function of the depth d.

(b) Complete the table below. Show all of your work in the space below.

Depth (# of feet)	**Pressure (psi)**
0	
20	
40	
60	
80	
100	

(c) Make a graph of the function. Make sure you label your axes with the correct units.



(d) Describe in words why this is a function.

Celsius and Fahrenheit

Many functions can be determined by performing experiments in which measurements are taken to study the relationship between variables. Today you will gather data in class.

There are two containers of water, one with hot and one cold. You will need ten students to conduct this experiment:

- One time keeper
- Four students at the hot water station: two will read the Celsius thermometer and two will read the Fahrenheit thermometer
- Four students at the cold water station: two will read the Celsius thermometer and two will read the Fahrenheit thermometer
- One recorder

The time-keeper will call “hot” or “cold” each minute (See table below). Everyone should keep quiet so the time keeper can be heard.

When “hot” is called, two students at the hot water station will read the Celsius thermometer and agree on a reading they will give to the recorder. At the same time the other two students at the hot water station will agree on a Fahrenheit reading to give to the recorder. Two readers are used for each reading so that reading and recording errors can be reduced. When “cold” is called, the same approach is used. Readings for the hot and cold will alternate to give the recorders a chance to get both measurements.

1. Once the experiment has been completed, the recorders will share the data for the Celsius and Fahrenheit readings.

Table 1 – Temperature Readings

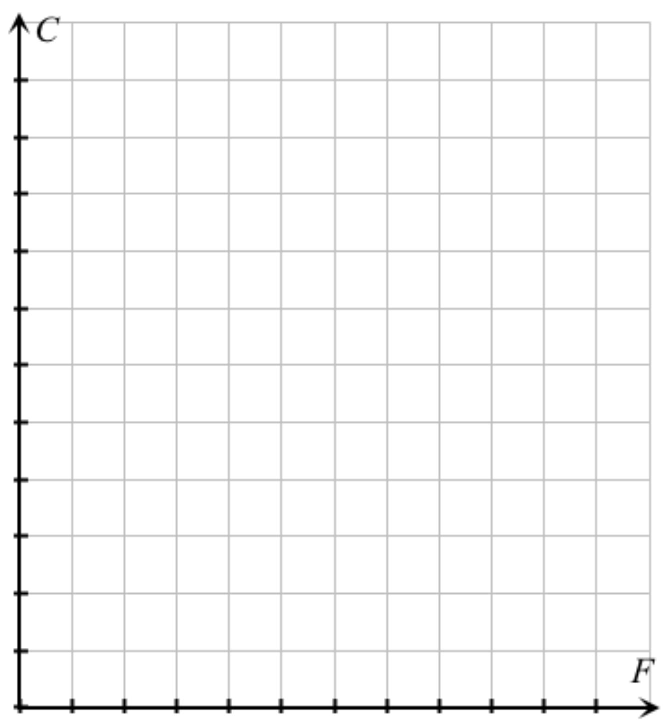
Minute	Hot or Cold	Reading °C	Reading °F
0	Hot		
1	Cold		
2	Hot		
3	Cold		
4	Hot		
5	Cold		
6	Hot		
7	Cold		
8	Hot		
9	Cold		

2. Complete Table 2 below. Place Fahrenheit values in the domain, listing them in increasing order. Then fill in the corresponding Celsius values in the range.

Table 2 – Temperature Data

°F	°C

3. Graph the data in Table 2 below. Label the axes.



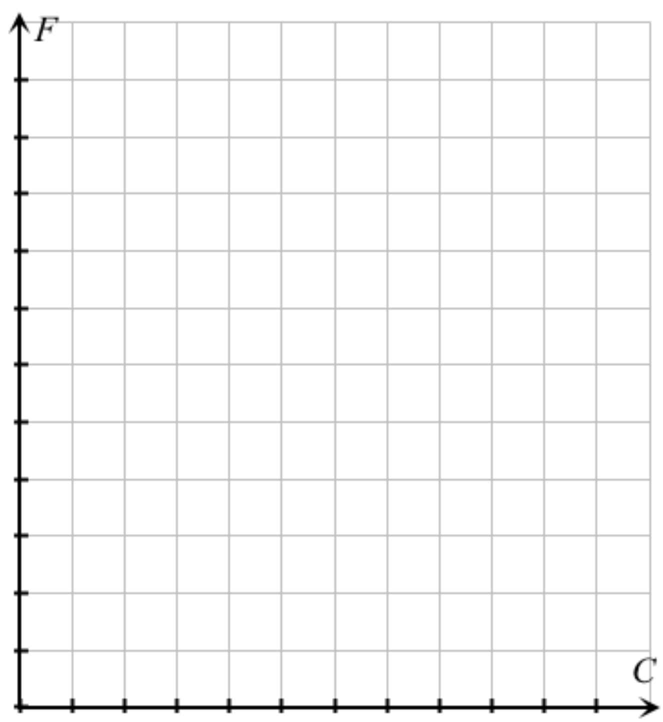
4. Do you notice any trend in the graph above? Explain.

5. Complete Table 3 using the data from Table 1. Place Celsius values in the domain, listing them in increasing order. Then fill in the corresponding Fahrenheit values in the range.

Table 3 – Temperature Data

°C	°F

6. Graph the data in Table 3.



7. Do you notice a trend? Explain.

8. Are the graphs in questions 3 and 6 the same? Does changing the domain and range make a difference?

The Raven and the Jug

This experiment is based on Aesop's Fable: The Crow and the Pitcher.

> A Crow, half-dead with thirst, came upon a Pitcher which had once been full of water; but when the Crow put its beak into the mouth of the Pitcher he found that only very little water was left in it, and that he could not reach far enough down to get at it. He tried, and he tried, but at last had to give up in despair. Then a thought came to him, and he took a pebble and dropped it into the Pitcher. Then he took another pebble and dropped it into the Pitcher. Then he took another pebble and dropped that into the Pitcher. Then he took another pebble and dropped that into the Pitcher. Then he took another pebble and dropped that into the Pitcher. Then he took another pebble and dropped that into the Pitcher. At last, at last, he saw the water mount up near him, and after casting in a few more pebbles he was able to quench his thirst and save his life.

Source: http://ancienthistory.about.com/library/bl/bl_aesop_crow_pitcher.htm.

1. You will be given a jar half filled with water, a set of marbles, and a ruler. You will study what happens to the water level as you add marbles to the jar. Record your data in the table below.

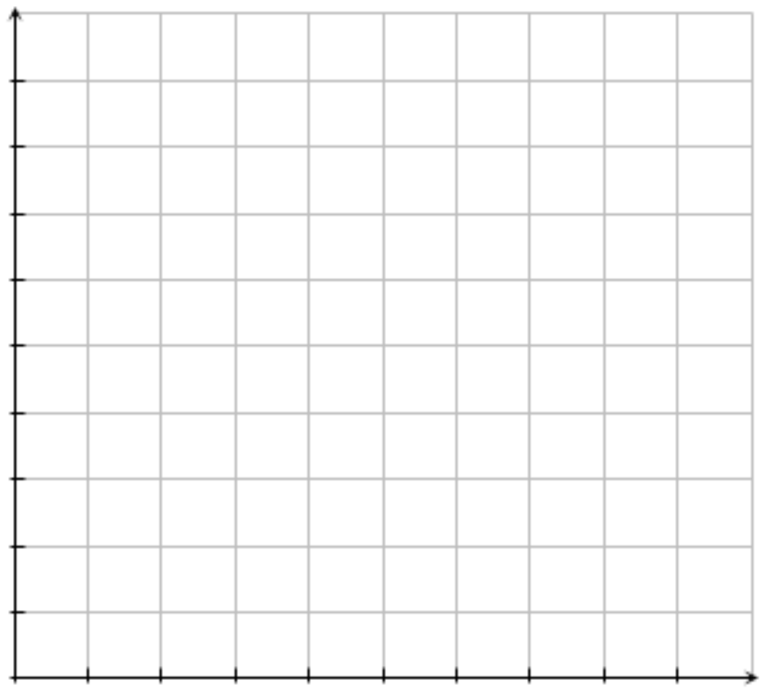
   Your group should decide how many marbles to add each time in order to make the change in the water level noticeable.

# of Marbles	Water Level

2. Identify the independent variable: ____________________________________

3. Identify the dependent variable: ____________________________________

4. Make a graph of the data. Label and scale the axes accordingly.



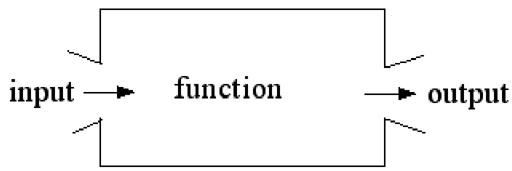
5. Describe any trends you see in the graph.

6. Is the relationship between the number of marbles and the height of the water a function? Explain.

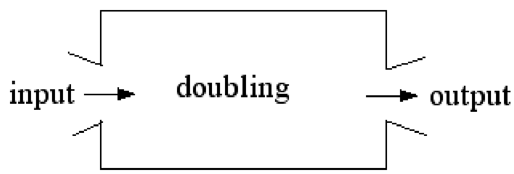
<u>Note:</u> This activity is adapted from one found in *Algebra Experiments I: Exploring Linear Functions* by Mary Jean Winter and Ronald J. Carlson, Menlo Park, CA: Addison-Wesley, 1993.

Function Machines

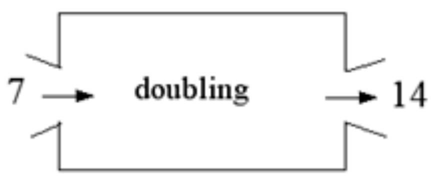
Here is a function machine: It takes one number as an **input** and produces another number as the **output**. A **function** is a rule that tells the machine what output to produce for any input.



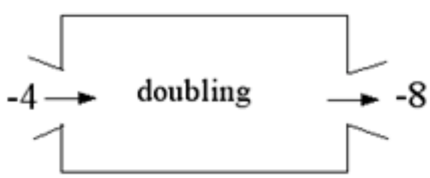
Example 1: The doubling function. The rule here is that the output is twice the input.



Here are some examples of how this function works.



When the input is 7 the output is 14.
We can say, "Doubling 7 gives you 14."
We can write "doubling(7) = 14"
(doubling OF 7 equals 14)



When the input is –4 the output is –8.
We can say, "Doubling –4 gives you –8."
We can write "doubling(–4) = –8"
(doubling OF –4 equals –8)

Questions on Example 1:

1. If the input of the doubling function is 5, what is the output? ________
2. Doubling (6) = ______?
3. Doubling (–9) = ______?
4. When the input of the doubling function is a positive number, what kind of number (positive or negative) is the output? __________
5. When the input of the doubling function is a negative number, what kind of number (positive or negative) is the output? __________

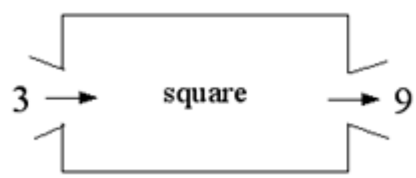
6. Another way of describing the doubling function is that the output is _____ times the input.
7. What is the domain of the doubling function? What is the range?

8. Make a table of values for the doubling function. Chose your own values for the last three inputs.

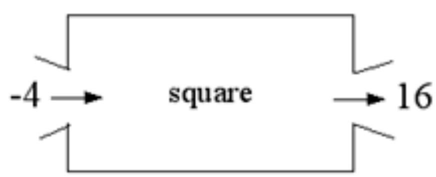
Input	**Output**
–9	
–4	
0	
3	
6	
7	

Example 2: The square function. The rule here is that the output is found by multiplying the input by itself.

Here are some examples of how this function works.



When the input is 3 the output is 9.
We can write "square(3) = 9"
(The square OF 3 equals 9)



When the input is –4 the output is –16.
We can write "square(–4) = 16"
(The square OF –4 equals 16)

Questions on Example 2:

9. If the input of the square function is 5, what is the output? ________
10. Square (6) = ______?
11. Square (–9) = ______?
12. When the input of the square function is a positive number, what kind of number (positive or negative) is the output? __________

13. When the input of the square function is a negative number, what kind of number (positive or negative) is the output? __________
14. If the input of the square function is zero, what is the output?_______
15. If *x* is the input of the square function, what is the output? ______
16. What is the domain of the square function? What is the range?

17. Make a table of values for the square function. Chose your own values for the last three inputs.

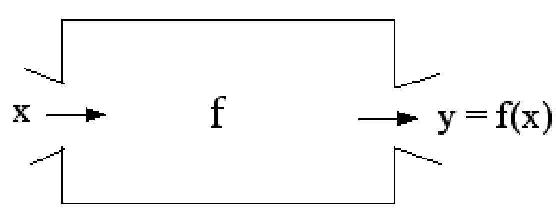
Input	Output
–9	
–4	
0	
3	
6	

Function Notation:

We have observed that functions may be given names, such as "doubling" or "square." Sometimes we just use a letter of the alphabet for the function name. Since the word "function" begins with the letter "*f*" we most often use *f*. Sometimes we use *g* or *h* since they come after *f* in the alphabet.

Since the input and output of the function may take on many values they are **variables.** The input is sometimes called the **independent variable** and the output the **dependent variable**. (Here's how to remember this: the output **depends** upon the input.)

The letter ***x*** is often use for the input (independent) variable. The letter ***y*** is used for the output (dependent) variable. Thus, functions in general may be represented by this machine.



Example 3

Functions may be defined by algebraic rules. Suppose *f(x)*, *g(x)*, and *h(x)* are defined as follows:

$$f(x) = x + 10 \qquad g(x) = 3x \qquad h(x) = 4x - 5$$

Questions on Example 3:

18. For the function *f*, when the input is 6, the output is _____.
19. For the function *g*, when the input is –5, the output is ______.
20. For the function *h*, when the input is 2, the output is ______.
21. Find *f*(–3).
22. Find *g*(7).
23. Find *h*(7).
24. Fill in the tables below:

x	***f*(*x*)**
–3	
–2	
–1	
0	
1	
2	
3	

x	***g*(*x*)**
–3	
–2	
–1	
0	
1	
2	
3	

x	***h*(*x*)**
–3	
–2	
–1	
0	
1	
2	
3	

25. Describe any patterns you see in the table above.

26. Does *f*(*x*) mean "*f* times *x*"? Explain.

Introduction to Function Notation

Jane works as a travel agent. Every week she gets paid $900 (her base salary) plus $100 for each cruise that she books (her commission). We can use a function to describe the relationship between her weekly salary and the number of cruises that she books.

1. Fill in the blanks below:

Jane's __________________________ depends on __________________________________, and every

number of _____________________________ gives only one __________________________________.

Therefore, we can say her ______________________ is a function of ____________________________.

2. Write a recursive rule to describe Jane's weekly salary.

3. Let **S** be her weekly salary, and **c** be the number of cruises she books. Write an explicit rule to model Jane's weekly salary based on the number of cruises she books.

4. Jane's weekly salary is a function of number of cruises she books. We can use a special notation called function notation to label it. Instead of writing ***S*** alone as we did in the explicit rule, we write ***S*(*c*)**. Write ***S*(*c*)** below.

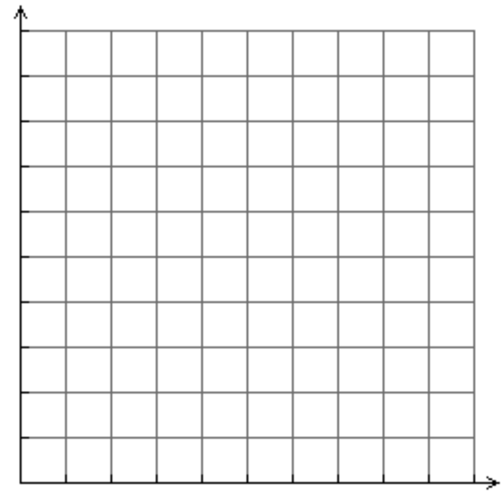
5. Let's look at the parts of that notation, ***S*(c)**, which is read "**S of c**." Fill in the table below.

S is the:	**c** is the:
S is the ________________variable	**c** is the ________________variable
The parentheses hold the place for the input value, they do not signify multiplication!	

6. Complete the table below.

c	***S*(*c*)**
0	
1	
2	
3	
4	
5	

7. Graph the function on the axes below. Label the axes appropriately.



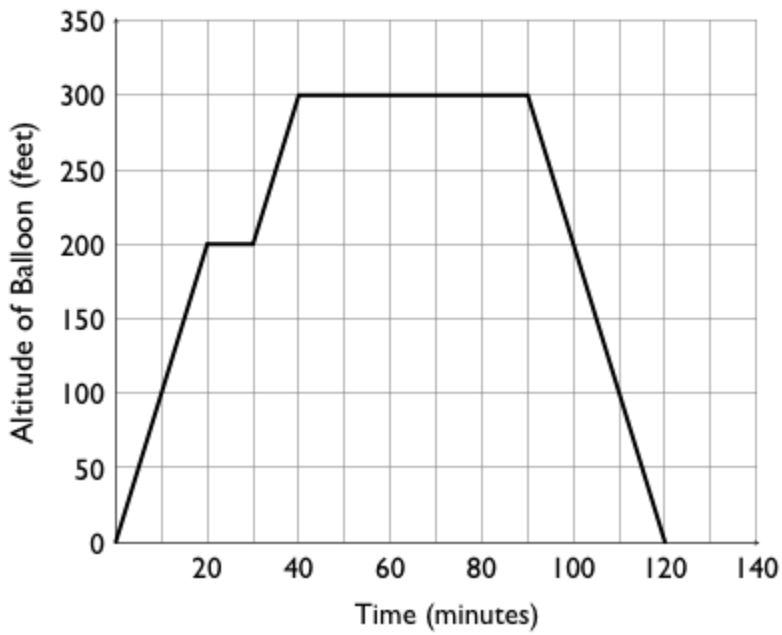
8. Fill in the mapping diagram below.

0	1100
1	900
2	1400
3	1000
4	1200
5	1300

9. What are the domain and range of this function?

Hot Air Balloon

The graph below shows the altitude during a hot air balloon ride with Berkshire Balloons. The altitude of the hot air balloon is a function of time.



1. Find the domain and range of the function graphed above.

   Domain: Range:

2. Find $f(30)$ and explain what it means in the context of the problem.

3. If $f(x) = 100$, find all values of x and explain what they mean in the context of the problem.

4. When is the balloon at 200 feet?

5. For how long are you flying at an altitude at or above 200 feet?

6. If $f(x) = 300$, find all values of x.

Function Applications – Highway Driving

Leon went to his Grandma's home for Thanksgiving. His dad set the cruise control at 60 miles per hour while they drove on the highway. They travelled on the highway for 4 hours. Create a function that models the distance traveled on the highway, ***d*** (in miles), after they have driven on the highway for ***t*** hours.

(a) Independent variable:

(b) Dependent variable:

(c) Write the equation for this function.

(d) Use function notation to express the function.

(e) We can say ____________________________ is a

function of ______________________________.

(f) Find the distance Leon's family travelled after driving for 3.2 hours on the highway. Use function notation.

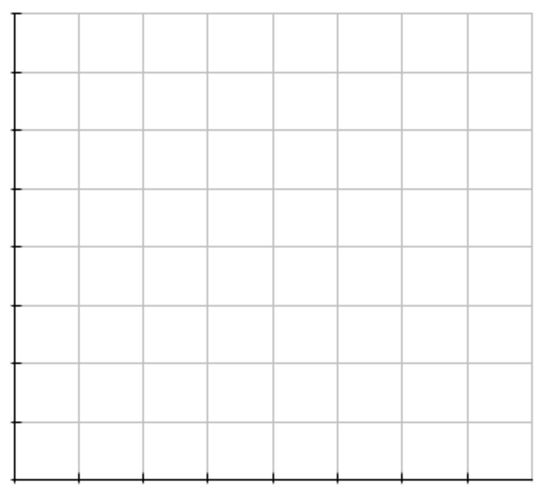
(g) Find the time it took for Leon's family to travel 175 miles on the highway.

(h) What are the domain and range of this function?

(i) Complete the table below.

Input	**Output**

(j) Graph the function on the axes below. Scale and label the axes.



(k) Identify the shape of this graph using the Parent Function Reference Sheet.

Function Applications – Highway Driving

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(b) Dependent variable:

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(d) Use function notation to express the function.

(e) We can say ____________________________

is a function of ______________________.

(f) Find the distance Leon's family travelled after driving for 3.2 hours on the highway. Use function notation.

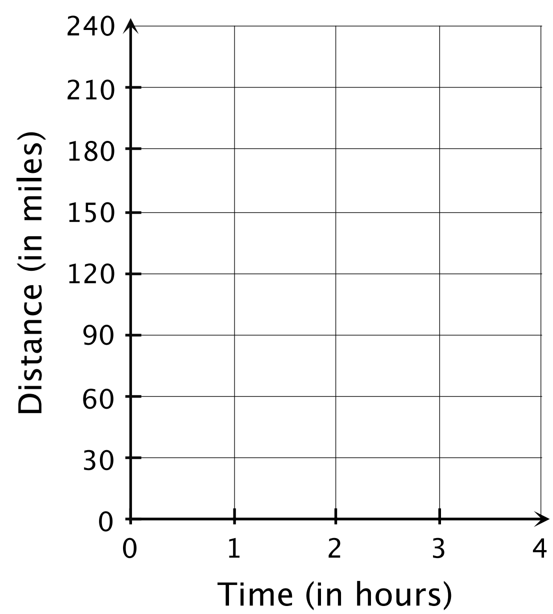
(g) Find the time it took for Leon's family to travel 175 miles on the highway.

(h) What are the domain and range of this function?

(i) Complete the table below.

Input Time (hours)	**Output** Distance (miles)

(j) Graph the function on the axes below.



(k) Identify the shape of this graph using the Parent Function Reference Sheet.

Function Applications – Travel Time

Leon's relatives all had to travel 150 miles to Grandma's home, but they each travelled different amounts of time. Create a function that models the travel time t (in hours) based on the velocity v (in miles per hour) that a relative travelled.

(a) Independent variable:

(b) Dependent variable:

(c) Write the equation for this function:

(d) Use function notation to express the function:

(e) We can say ______________________________ is a

function of ______________________________.

(f) Find the travel time of Uncle Jim who travelled at 60 mph. Use function notation.

(g) Find the velocity of Cousin Tina who took 4 hours to get to Grandma's.

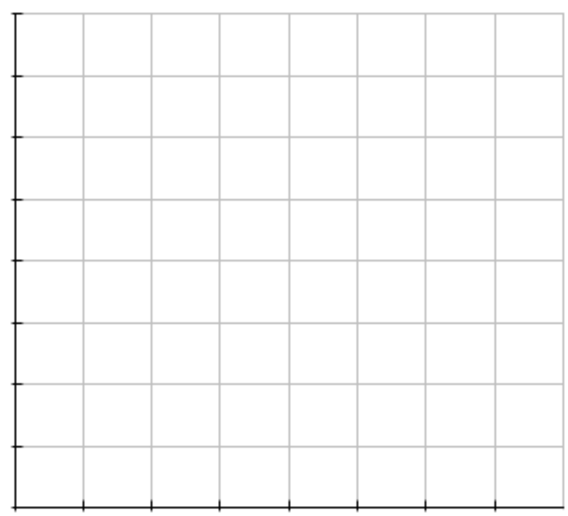
(h) What are the domain and range of this function?

(i) Identify the shape of this graph using the Parent Function Reference Sheet.

Complete the table below:

Relative	**Velocity (mph)**	**Travel Time (hours)**
Great Grandpa	30	
Aunt Violet	45	
Uncle Harry	60	
Cousin Will	75	

Graph the function on the axes below. Scale and label the axes.



Function Applications – Height of a Ball

Ben's free throw follows a curved path. It goes up and then comes back down. The height of the ball ***h*** (in meters) at time ***t*** (in seconds) is given by the equation $h = -5t^2 + 10t + 1$.

(a) Independent variable:

(b) Dependent variable:

(c) Use function notation to express the function:

(d) We can say ______________________________ is a

function of ________________________________.

(e) Find the height of Ben's shot after 2.5 seconds. Use function notation.

(f) Find the time it takes for Ben's shot to be 10 meters above ground.

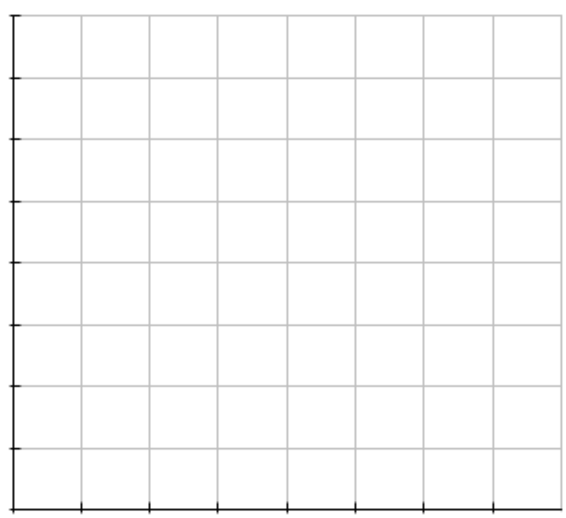
(g) What are the domain and range of this function?

(h) Describe the shape of this graph. Use the Parent Function Reference Sheet.

Complete the table below:

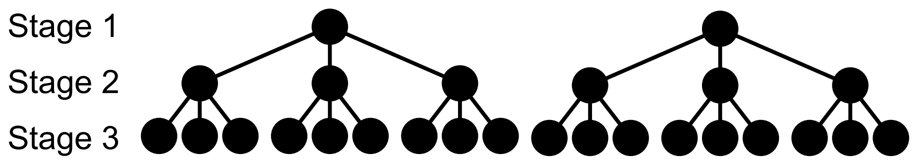
Time (seconds)	**Height (meters)**
0	
.4	
.8	
1.0	
1.2	
1.6	
2.0	

Graph the function on the axes below.
Scale and label the axes.



Function Applications – Phone Tree

A city-wide soccer club has a phone tree to keep members informed of cancellations due to bad weather. At the first stage of the calling process, 2 leaders are informed. In stage 2, each person who was informed in stage 1 calls 3 players who are not informed. In stage 3, each person who was informed in stage 2 calls 3 players who are not informed. This pattern repeats and ends after Stage 5. The number of players informed at each stage, *P*, is a function of the stage number *x*.



(a) Independent variable:

(b) Dependent variable:

(c) The equation $P = 2 * 3^{x-1}$ represents the number of players *P* informed in each stage *x*. Express the function using function notation.

(d) We can say ______________________________ is

a function of ______________________________.

(e) Write a sentence to express the meaning of the following equation. $P(4) = 54$

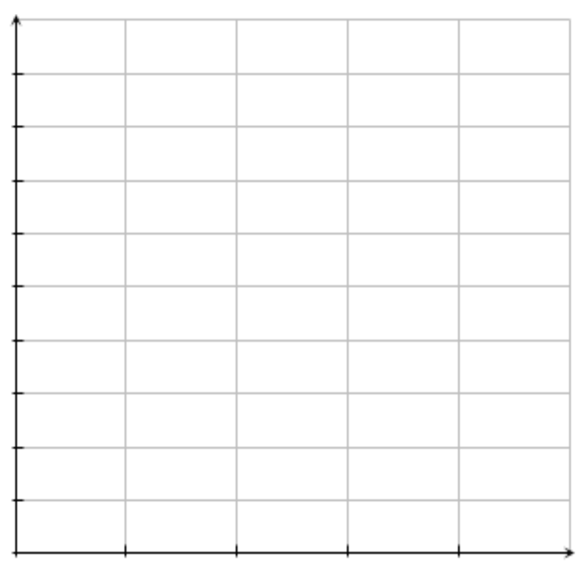
(f) What are the domain and range of this function?

(g) Describe the shape of this graph. Use the Parent Function Reference Sheet.

Complete the table below:

Input Stage of Phone Tree	**Output** # of Players informed
1	2
2	6
3	
4	
5	

Graph the function on the axes below.
Scale and label the axes.



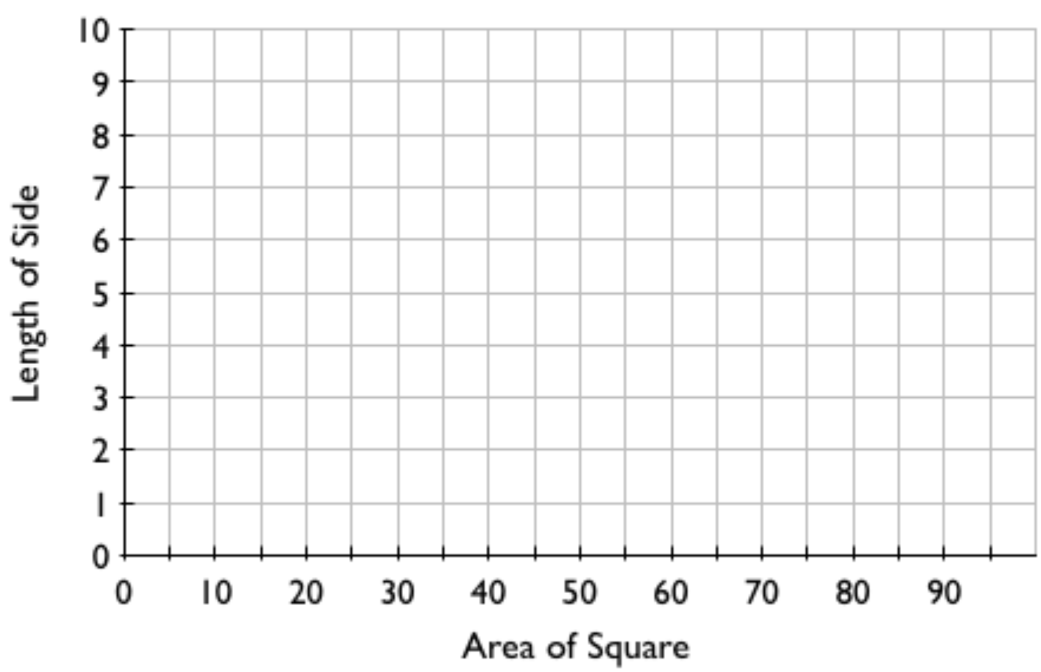
Geoboard Squares

1. Use a five-by-five geoboard (or dot paper) and find as many squares of different sizes as you can.

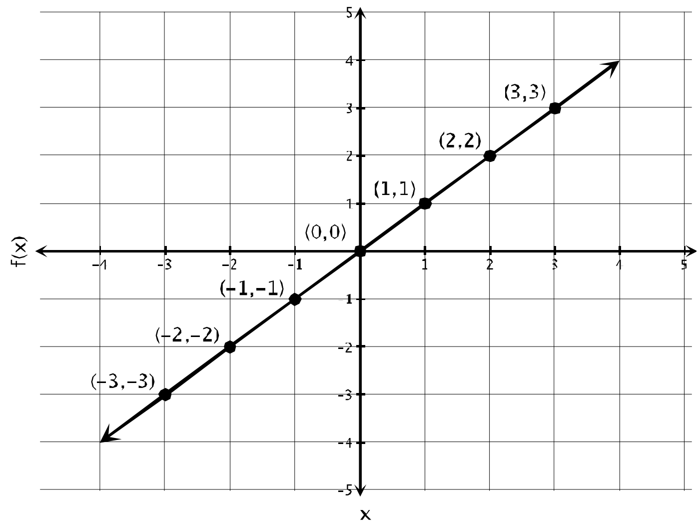
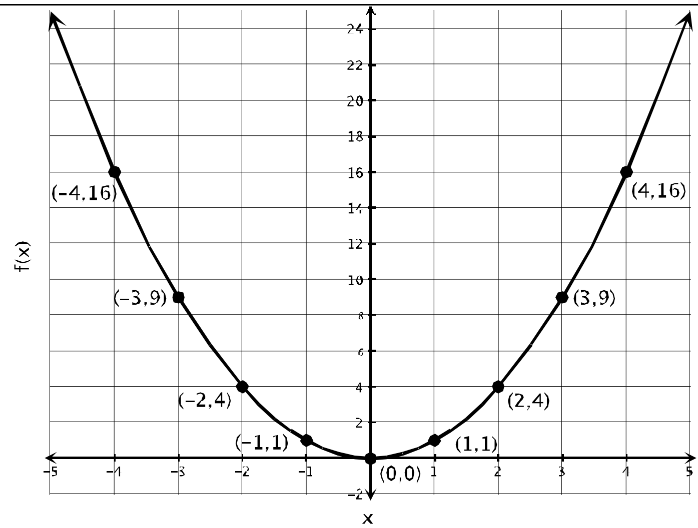
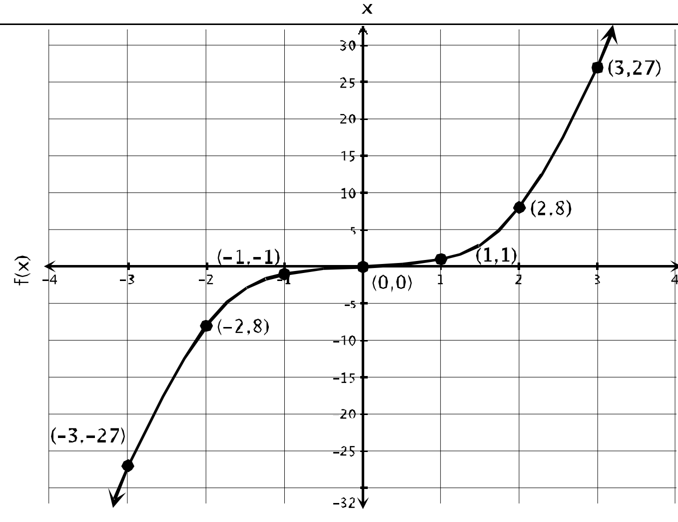
2. For each square you find on the geoboard, record the area and the length of a side (to the nearest 0.1 unit) in the table.

Area of Square (square units)	**Length of Side (units)**

3. Graph the length of a side as a function of the area of the square.



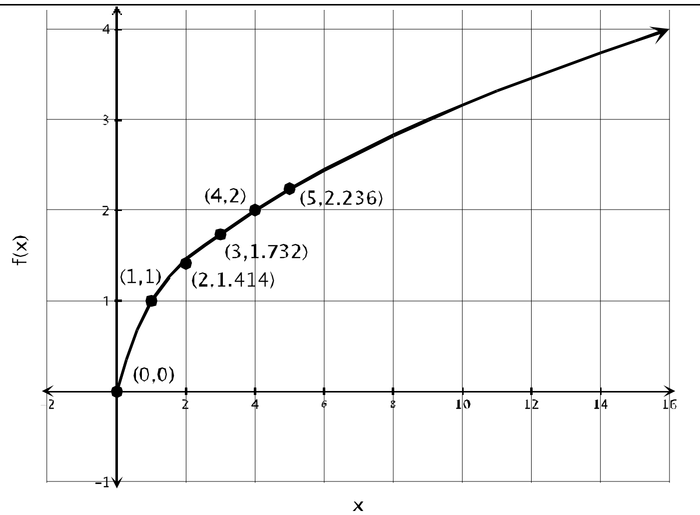
4. Describe the shape of this function. Use the Parent Function Reference Sheet.

<table>
<tr><th>Function</th><th>Table</th><th>Graph</th></tr>
<tr><td>Linear
$f(x) = x$</td><td>
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>-3</td><td>-3</td></tr>
<tr><td>-2</td><td>-2</td></tr>
<tr><td>-1</td><td>-1</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>2</td></tr>
<tr><td>3</td><td>3</td></tr>
</table>
</td><td>
</td></tr>
<tr><td>Quadratic
$f(x) = x^2$</td><td>
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>-3</td><td>9</td></tr>
<tr><td>-2</td><td>4</td></tr>
<tr><td>-1</td><td>1</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>9</td></tr>
</table>
</td><td>
</td></tr>
<tr><td>Cubic
$f(x) = x^3$</td><td>
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>-2</td><td>-8</td></tr>
<tr><td>-1</td><td>-1</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>8</td></tr>
</table>
</td><td>
</td></tr>
</table>

Square Root

$f(x) = \sqrt{x}$

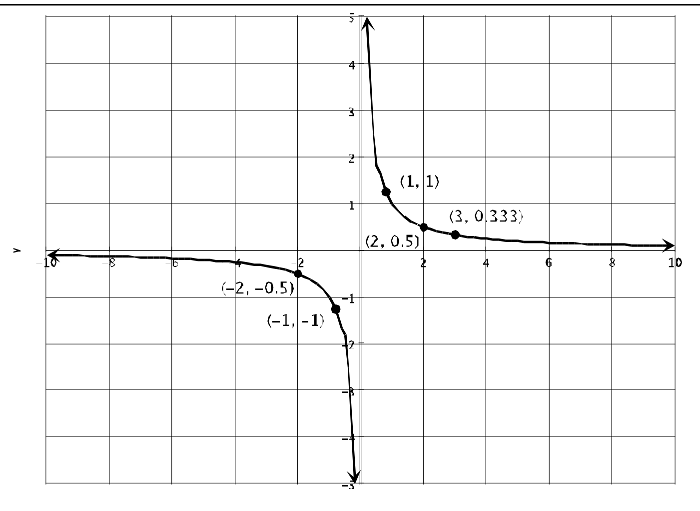
x	f(x)
0	0
1	1
2	1.414
3	1.732
4	2
5	2.236



Reciprocal

$f(x) = \frac{1}{x}$

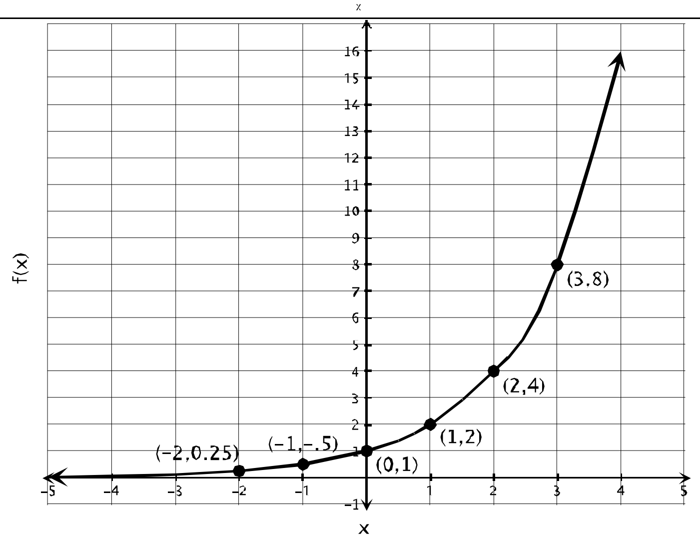
x	f(x)
-2	-0.5
-1	-1
0	undefined
1	1
2	0.5
3	0.333



Exponential (increasing)

$f(x) = 2^x$

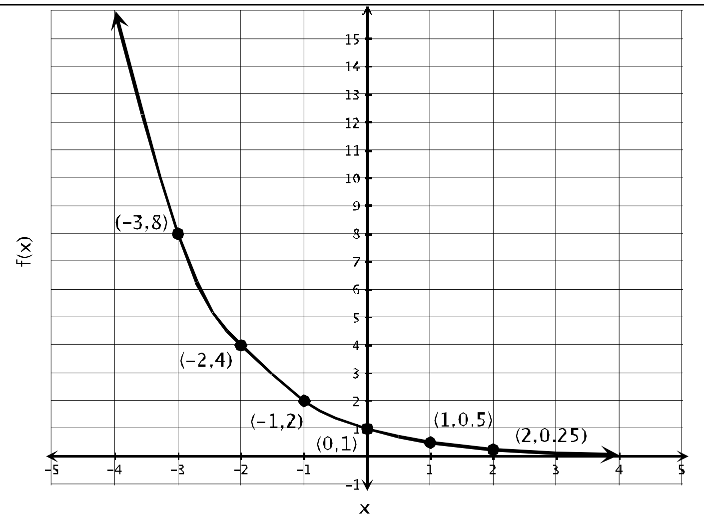
x	f(x)
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8



Exponential (decreasing)

$f(x) = 0.5^x$

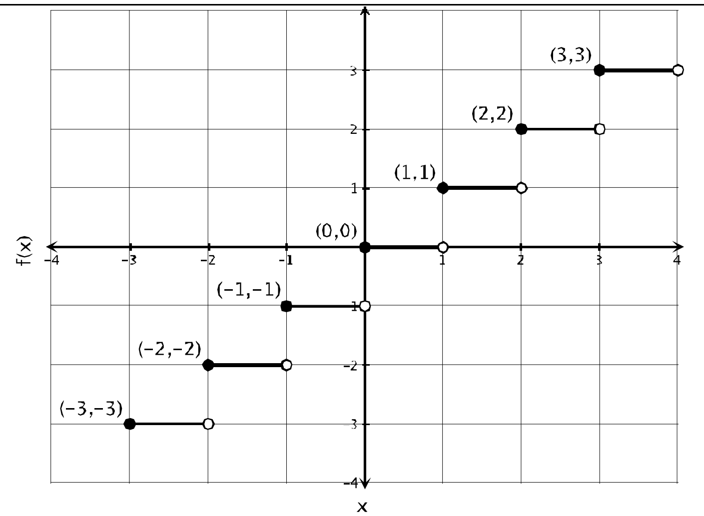
x	f(x)
-3	8
-2	4
-1	2
0	1
1	0.5
2	0.25



Step (floor)

$f(x) = \lfloor x \rfloor$

x	f(x)
--3	-3
-2	-2
-1	-1
0	0
1	1
2	2
3	3



Absolute Value

$f(x) = |x|$

x	f(x)
-3	3
-2	2
-1	-1
0	0
1	1
2	2
3	3