**Unit 1: Investigation 4 (2 days)**

**Geometric Sequences**

***CCSS: F-BF 1, F-BF 2***

**Overview**

In this investigation students will examine geometric sequences through real world applications.

**Assessment Activities**

 **Evidence of Success: What Will Students Be Able to Do?**

Find recursive rules, calculate terms of geometric sequencesc and explain the difference between an arithmetic sequence and a geometric sequence.

 **Assessment Strategies: How Will They Show What They Know?**

**Exit Slip 1.4** asks students to find terms in a geometric sequence, identify the recursive rule, represent the sequence using a table and graph, and use the sequence to solve a problem in context.

**Journal Entry** asks students to describe the difference between a geometric and arithmetic sequence and to provide a real-world example of each type of sequence.

**Launch Notes**

Discuss that geometric sequences arise in many real-world situations in finance and nature. **Activity 1.4.1 Doubling Your Money** provides students an opportunity to explore the growth of an investment over time. Allow students time to explore the problem and provide direction as needed. When introducing geometric sequences, stress that geometric sequences are similar to arithmetic sequences in that we do the same thing to each term to get the subsequent term. Students discover that instead of adding a fixed amount to each term as is done in arithmetic sequences, geometric sequences require that we multiply each term by the same *factor* to get the subsequent term.

**Closure Notes**

Conduct a whole classroom discussion on the characteristics of a geometric sequence and how they differ from arithmetic sequences. **Exit Slip 1.4.1** may be used in advance of or at the end of the class discussion.

**Teaching Strategies**

1. In **Activity 1.4.1 Doubling Your Money**, students explore an investment which behaves like a geometric sequence. Distribute the activity to students allowing them time to explore the problem. Depending on student progress, encourage students to construct a table that shows the growth from year to year. There are multiple ways that students can solve this problem. Have students work individually or in pairs and then have students share their answers and strategies with the class. If no student (or group of students) chooses to use the graphing calculator, you may want to review how to use the recursive function of the home screen to keep multiplying by 1.08.

A nice extension to this activity is a discussion of the *Rule of 72*. This rule states that the amount of time it takes for an investment to double is approximately 72 divided by the interest rate. In Activity 1.4.1, the Rule of 72 produces 72 ÷ 8 = 9 years. Here is an online calculator that compares the estimate with the “real” answer: <http://www.moneychimp.com/features/rule72.htm>.

**Group Activity**

**Activity 1.4.1 Doubling Your Money** can be done by students individually or in pairs. Individual students or pairs of students can then share their strategies of how they solved the problem with the class

**Differentiated Instruction (For Learners Needing More Help)**

For students who struggle with the concept of percent increase, break down the percent calculation into finding the amount of interest earned each year and then the new value at the end of the year. Help students to see that an 8% increase is obtained by multiplying by 1.08.

**Differentiated Instruction (Enrichment)**

As an additional extension of **Activity 1.4.1 Doubling Your Money**, students may explore ways to calculate the rules for tripling (114) and quadrupling (144) their money. Information can be found at <http://allfinancialmatters.com/2007/05/14/the-rule-of-72-114-and-144/>

1. In **Activity 1.4.2 Applications of Geometric Sequences**, students explore geometric sequences, identify their recursive rule, and represent sequences using tables and graphs. At the end of the activity students are asked to describe the difference between an arithmetic and geometric sequence. If possible, encourage students to create the geometric sequences using a graphing calculator or Microsoft Excel. If students use Excel, show students how to plot the sequences so they can see the non-linear behavior. The relationship between geometric sequences and exponential functions will be developed and expanded on in Unit 7.

**Differentiated Instruction (Enrichment)**

Ask students to identify the explicit rules for certain geometric sequences explored in this investigation.

1. In **Activity 1.4.3 More Geometric Sequences**, students solve a variety of problems by creating and exploring geometric sequences. This activity contains less scaffolding then previous activities and serves as a wrap up to the investigation.

Problem 4 in **Activity 1.4.3 More Geometric Sequences** introduces a derivation of the “penny problem”. In this problem, students explore a payment plan in which a penny is paid on the first day, and each subsequent day the payment doubles. Here is a site showing what collections of pennies look like: <http://www.kokogiak.com/megapenny/default.asp>.

Another possible extension is the story of the Devil and Daniel Webster. (Note that some portions of this activity are not arithmetic or geometric sequences.) The problem comes from NCTM’s *Illuminations* collection of lessons and can be found at: <http://illuminations.nctm.org/LessonDetail.aspx?id=L288>.

All problems in Activity 1.4.3 and the Devil and Daniel Webster activity should be supplemented with graphing calculators or Microsoft Excel.

**Journal Prompt**

In your own words describe the difference between an arithmetic sequence and a geometric sequence. Give an example of each.

**Resources and Materials**

* **Activity 1.4.1** – Doubling Your Money
* **Activity 1.4.2** – Applications of Geometric Sequences
* **Activity 1.4.3 –** More Geometric Sequences
* **Exit Slip 1.4.1** – Population Growth
* Graphing Calculators
* Microsoft Excel