**Unit 4: Linear Functions**

**(6 Weeks)**

**UNIT OVERVIEW**

Students start Unit 4 by exploring the distinction between linear and nonlinear behavior, and then focus on learning about linear functions. Throughout Unit 4, students derive linear models of real-world situations in order to analyze situations, make predictions or solve problems. Analyzing situations often takes the form of identifying the real world meaning of the slope and the *x*- and *y*-intercepts of a linear model. Making predictions involves evaluating models for a given independent variable (given *x* find *y*), and solving equations for the independent variable given the dependent variable (given *y* find *x*). Problem solving occurs through the use of various representations: algebraic, tabular, graphic and numeric.

In the first investigation students begin to develop the concept of constant rate of change by examining the data generated by a motion detector as displayed in a time-distance graph. Students will understand that “walking steadily” creates a straight-line graph, whereas “speeding up” or “slowing down” creates a graph that is non-linear. Decreasing the distance from the starting place (the motion detector) will produce a graph that decreases as one reads from left to right. Conversely, increasing the distance from the starting place will create a graph that increases as one reads from left to right.

Investigation 2 provides practice recognizing and describing functions from data tables and graphs in various contextual situations. Students identify the characteristics of a linear function, investigate the role of slopes and *y*-intercepts in the graphs of functions and relate this information to the context of various problems. Students create graphs by hand and with the graphing calculator. They engage in activities that highlight the capability of linear functions to model a wide range of real world relationships. A quiz is suggested after this investigation.

The third investigation provides an in-depth focus on slope as constant additive change, a definitive attribute of linear functions. Students calculate the slope from data in tables and graphs. They identify and interpret the slope from real-world linear situations as the constant rate of change in the dependent variable compared to the change in the independent variable.

The fourth investigation fully develops the $f\left(x\right)=mx+b$ and $y=mx+b$ forms, which students intuitively used to model a variety of real world situations and to define patterns explicitly as far back as Unit 1. Students explore the results of how changing the two parameters *m* and *b* changes the graph of a linear function. They will discover that changing the *y*-intercept causes a vertical shift in the graph, that the sign of the slope determines whether the graph is increasing or decreasing, and that the magnitude of the slope affects the steepness of the graph. Students will be able to graph a function given in slope-intercept form not only by making a table of values, but also by first plotting the *y*-intercept and then one or more additional points using the slope. Students will be able to find the slope-intercept equation of a line from a graph, table or real-world scenario, thus reinforcing the multi-representational approach. As in other investigations, students have opportunities to use what they are learning to solve a variety of contextual problems. To complete the investigation students will discover the relationships of the slopes of parallel lines and of perpendicular lines. At this point, students may be assessed by a Mid-Unit Test.

Investigation 5 introduces the standard form of the equation of a line. Equations in standard form are graphed two ways: by finding the *x-* and *y-* intercepts and by transforming the standard equation to slope-intercept form. Students will also investigate direct variation problems, which are modeled by the family of linear functions that have a *y*-intercept of 0. A quiz is suggested after this investigation.

The sixth investigation develops the point-slope form of the line. Students can identify a point and the slope from the equation and can write an equation in point-slope form given a point and slope or given two points. Students transform equations written in point-slope form into slope- intercept or standard form. Contextual activities and real world situations are used to introduce the forms of linear equations as well as to extend students’ knowledge of linear functions. By the end of this investigation, students will choose which of the three forms of linear equations is most advantageous for solving a particular problem.

The performance task assesses the ability of students to distinguish between linear and nonlinear relationships and then make predictions based on the linear functions. The End-of-Unit Assessment checks for mastery of the key concepts and skills in the unit.

**Essential Questions**

* What is a linear function?
* What are the different ways that linear functions may be represented?
* What is the significance of a linear function’s slope and *y*-intercept?
* How may linear functions model real world situations?
* How may linear functions help us analyze real world situations and solve practical problems?

**Enduring Understandings**

Linear functions are characterized by a constant average rate of change (or constant additive change).

**Unit Contents**

Investigation 1: What Makes a Function Linear? (2 days)

Investigation 2: Recognizing Linear Functions from Words, Tables and Graphs (4 days including Quiz on Investigations 1 and 2)

Investigation 3: Calculating and Interpreting Slope (4 days)

Investigation 4: Effects of Changing Parameters of an Equation in Slope-Intercept Form (4 days)

Mid-Unit Test (2 days including review)

Investigation 5: Forms of Linear Equation (4 days including Quiz on Investigation 5)

Investigation 6: Point-Slope Form of Linear Equations (4 days)

Performance Task: Linear Models (2 days)

End of Unit Test (2 days including review)

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Standards Overview**

* Interpret functions that arise in applications in terms of the context
* Analyze functions using different representations
* Construct and compare linear [and exponential] models and solve problems
* Interpret expressions for functions in terms of the situation they model

**Standards with Priority Standards in Bold**

F-IF 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.\*

F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

1. Graph linear ...functions and show intercepts..

F-IF 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

**F-LE 1.** Distinguish between situations that can be modeled with linear functions [and with exponential functions].

a. Prove that linear functions grow by equal differences over equal intervals...

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another....

F-LE 2. Construct linear ... functions, including arithmetic ... sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

**F-LE 5. Interpret the parameters in a linear ... function in terms of a context.**

**Vocabulary**

Constant Additive Change

Convex Polygon

Dependent Variable

Direct Variation

Independent Variable

Initial Value

Linear Function

Linear Models

Magnitude

Nonlinear Function

Parameters

Piecewise Function

Point-Slope Form

Rate of Change

Slope

Slope-Intercept Form

Standard Form

Unit Rate

Velocity

*x*-intercept

*y*-intercept

**Assessment Strategies**

**Performance Task: Linear Models**

The Unit 4 performance task *Linear Models* requires students to investigate population growth or population decline. Students will be expected to demonstrate and apply their understanding of key concepts related to linear functions. Students will examine and compare the accuracy of their linear population models and present their findings in class.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* Unit 4 Investigations 1 & 2 Quiz
* Mid-Unit Test
* Unit 4 Investigation 5 Quiz
* Unit 4 Test