**Unit 8: Investigation 3 (4 days)**

**SOLVING QUADRATIC EQUATIONS USING THE SQUARE ROOT PROPERTY**

***CCSS: 8-EE 2, A-REI 4***

**Overview**

Students will learn to apply the Square Root Property and the principle of “undoing” to solve equations of the form $a(x-h)^{2}+ k = y.$ They will then apply this skill to finding the

*x*-intercepts of parabolas given functions in vertex form and to solving application problems based on quadratic functions in vertex form.

**Assessment Activities**

 **Evidence of Success: What will Students Be Able to Do?**

* + Recognize the relationship between squares and square roots.
	+ Recognize and distinguish quadratic functions in standard form and in vertex form.
	+ Undo quadratic expressions to find solutions to equations.
	+ Solve equations of the form $a(x-h)^{2}+ k $ = constant.
	+ Find the *x*-intercepts of parabolas given functions in vertex form.

 **Assessment Strategies: How Will They Show What They Know?**

* **Journal Entry 1** has students explain the square root property.
* **Exit Slip 8.3.1** assesses students’ ability to solve two-step equations with the square root property.
* **Exit Slip 8.3.2** assesses students’ ability to find the *x*-intercepts of a parabola when the function is given in vertex form.
* **Journal Entry 2** has students explain the steps used to solve a quadratic equation by “undoing” operations.

**Launch Notes**

Start with the problem in **Activity 8.3.1 Fenway Park** that asks students to find how far a baseball travels after being hit by a batter in Fenway Park. You may want to project an image of Fenway Park for students to view. Students can discuss how to solve this problem. Some will suggest making a graph, which is a valid, practical method. Inform students that in this Investigation they will learn how to algebraically solve problems involving quadratic functions.

**Closure Notes**

Hold a class discussion about **Activity 8.3.6 Solving Quadratic Equations in Standard Form** that focuses on the two forms of quadratic functions. Emphasis should be placed on the square root property, undoing to solve equations, and finding *x-*intercepts and *y*-intercepts.

**Teaching Strategies**

1. The goal of this investigation is for students to be able to solve equations of the form $a(x-h)^{2}+ k = y$. We accomplish this by allowing students solve progressively more complex quadratic equations involving squaring.

In **Activity 8.3.2** **The Square Root Property**, studentssolve equations of the forms $x^{2}=y$. Students must understand that equations in this form have two possible outcomes. For example, for $x^{2}= 16$, the solutions are 4 and –4. So every positive number has two square roots, one positive and one negative. The positive one is called the *principal square root*. If *y* is negative there is no solution. In this activity, introduce the *Square Root Property***.**

**Square Root Property**: If $x^{2}= y$ and *y* is a positive number then $x=\sqrt{y} $or

$x=-\sqrt{y} $. This may be abbreviated $x=\pm \sqrt{y}$.

Explain to students that in some contexts we are only interested in the positive root; for example, when we find the side of a square whose area is 16 m2. In some cases we do not have a perfect square. When this occurs we often leave the principal square root in radical form (for example, $\sqrt{3}$). Students can use calculators to find decimal approximations for square roots. In this Unit, we are not requiring students to represent irrational square roots in simple radical form.

**Journal Entry 1**

Explain the Square Root Property and how it is used to solve equations. How is it similar to and different from the other properties of equality (Addition, Subtraction, Multiplication and Division) used to solve equations?

In **Activity 8.3.3 Solving Two Step Equations with the Square Root Property**,students solve two-step equations of the form $ax^{2}=y$ and $x^{2}+a=y$. Encourage students to recall their prior experience with equation solving. Remind them that they can undo operations by performing inverse operations.

**Differentiated Instruction (For Learners Needing More Help)**

For students who are struggling, it may be better to begin with an example in which the coefficient of the squared term is an integer. Another geometric example is to find the edge of a cube with known surface area using the formula $S=6e^{2}$.

After students have completed **Activity 8.3.3** and checked their work, you may use **Exit Slip 8.3.1** to assess their understanding.

**Differentiated Instruction (for Enrichment)**

**Activity 8.3.3** involves solving two-step equations using the Square Root Property. Towards the end of the activity students are to create flow charts showing how they solved the first three equations. Students might be able to create a flow chart for how they solved the fourth equation as well.

1. In **Activity 8.3.4 Multi-Step Equations with Square Roots**,students solve equations of the form $(x-h)^{2} = y$. For example, consider the equation $(x-7)^{2}=100$. Here we take the square root first, which gives two possible solutions, $x - 7 = \pm \sqrt{100}$. This may be written as two separate equations $x – 7=10$ and $x –7=- 10$. Solving each equation results in the solutions *x* = 17 and *x* = –3. Students should verify that both solutions satisfy the original equation by substituting for *x*. The first problem in **Activity 8.3.4** involves finding the square root of a perfect square. Subsequent examples may involve irrational numbers.

In **Activity 8.3.4**, students solve equations of the form $a(x-h)^{2}+ k = y$. To analyze the necessary equation solving steps, students will use a flow chart to represent the order of operations on the variable term and then reverse these steps to solve the equation.

For example, to evaluate $a(x-h)^{2}+ k = y$ using a flow chart:

Start with *x* 🡪 subtract *h* 🡪 square 🡪 multiply by *a* 🡪 add *k* 🡪 answer is *y*

Undoing: Given *y* to find *x*

Given *x* 🡨 add *h* 🡨 square root 🡨 divide by *a* 🡨 subtract *k* 🡨 start with *y*

Show students how this works for a specific example such as $4(x+3)^{2}+8=32$:

 $4(x+3)^{2}+8=32$ Given equation

 $4(x+3)^{2}=24$ Subtract 8 (or add –8) on both sides

 $(x+3)^{2}=6$ Divide by 4 (or multiply by ¼) on both sides

 $x + 3=\sqrt{6}$ or $x + 3=-\sqrt{6}$ Use the square root property

 $x= –3+ \sqrt{6}$ or $x= –3-\sqrt{6}$ Add –3 (or subtract 3) on both sides

The above solutions are *exact*. Students may also find decimal approximations to be

*x* ≈ -0.551 or *x* ≈ –5.449. Students should be able to justify each step in the process as they did in Unit 2.

**Differentiated Instruction (For Learners Needing More Help)**

Some students may prefer to continue using the flow chart, whereas others may quickly dispense with it. For the former group, you may want to allow them to use the flow chart for question 6 in **Activity 8.3.4** rather than requiring them to identify the properties used for each step.

**Journal Entry 2**

Describe the steps you would use to solve the equation $\frac{1}{5}(x+4)^{2}- 7=10$ and explain why you would do them in that order.

1. In **Activity 8.3.5 Finding *x*-Intercepts of Parabolas**, students apply the equation solving method presented in **Activity 8.3.4** to find the *x*-intercepts of a parabola given its equation in vertex form. Remind students that we find the *x*-intercept(s) of a graph by setting *y* = 0 in the equation. Present parabolas such as $y=(x+3)^{2}+4$ that do not intersect the *x*-axis have no real *x*-intercepts. Setting *y* = 0 and attempting to solve this equation for *x* leads to no real solutions.

In addition to giving students practice solving equations, **Activity 8.3.5** leads students to explore the relationship between the parameters *a* and *k* and the number of *x-*intercepts.

Now we are in a position to solve the initial problem of the Investigation in **Activity 8.3.1 Fenway Park**. See questions 9 and 10 in **Activity 8.3.5.** The given function was $y= -0.001(x-200)^{2}+ 44$. To find where the ball hits the ground, set the height, *y*, equal to 0, and then solve for *x*. The solutions are *x* = -9.8 and *x* = 409.8. In the context of this problem, the negative solution does not make sense (the baseball once hit does not go behind home plate). So we conclude that the ball will hit the ground about 410 feet away from home plate. Since it falls short of the fence in deep center field, the batter does not hit a home run.

1. In **Activity 8.3.6 Solving Quadratic Equations in Standard Form**, students use the knowledge they attained in Investigations 2 and 3 to find the *x*-intercepts of a quadratic function given in standard form. First they find the vertex and write the equivalent equation in vertex form. Then they set *y* = 0 and solve using the square root property and undoing. In effect, students now have the tools they need to solve any quadratic equation in standard form, although more traditional methods including factoring, completing the square, and the quadratic formula will be introduced in subsequent investigations.

**Group Activity**

Students may work in groups of three to complete **Activity 8.3.6**. Assign two problems from questions 1–6 to each student. Have each student find the *x*-coordinate of the vertex for the assigned problems (step 1). Pass papers to the right and check the previous work. Then each student finds the *y-*coordinate of the vertex and writes the equation in vertex form (steps 2 and 3). Again pass papers to the right and check the previous work. Now each student solves the equation using the method of undoing (step 4). Again pass papers to the write. Students now have their original equations and may check the solutions. Finally have the group work together on problem 7, which is more challenging since it involves fractions.

**Resources and Materials**

* **Activity 8.3.1** Fenway Park
* **Activity 8.3.2** The Square Root Property
* **Activity 8.3.3** Solving Two Step Equations with the Square Root Property
* **Activity 8.3.4** Multi-Step Equations with Square Roots
* **Activity 8.3.5** Finding *x*-intercepts of Parabolas
* **Activity 8.3.6** Solving Quadratic Equations in Standard Form
* **Exit Slip 8.3.1** Solving Two-Step Equations with Square Roots
* **Exit Slip 8.3.2** Finding *x*-intercepts
* Image of Fenway Park (for Activity 8.3.1) [http://commons.wikimedia.org/wiki/File:Fenway\_Park02.jpg](http://commons.wikimedia.org/wiki/File%3AFenway_Park02.jpg)
* Image of Parabolic Solar Cooker (for Activity 8.3.4) <http://solarcooking.wikia.com/wiki/Parabolic_cookers>
* Student Journals
* Graphing Calculators
* Bulletin Board