**Activity 1.7. 3 Properties of Isometries**

Consider the three basic isometries: translation, rotation, and reflection.

Work with a partner. Use Geogebra or coordinates if you need to check your responses.

**Part I.** The Geogebra file ctcoregeomACT1731.ggb may be used for experiments in part I.

Determine which transformations have these properties:

1. When segment $\overbar{AB }$is mapped onto segment $\overbar{A’B’},$ *AB* = *A’B’.*

 a. Is this a property of all translations? \_\_\_\_\_\_\_\_\_

 b. Is this a property of all rotations? \_\_\_\_\_\_\_\_\_\_\_

 c. Is this a property of all reflections? \_\_\_\_\_\_\_\_\_\_

1. When $∠$*ABC* is mapped onto $∠$*A’B’C’*, m $∠$ *ABC* = m $∠$ *A’B’C’*

 a. Is this a property of all translations? \_\_\_\_\_\_\_\_\_

 b. Is this a property of all rotations? \_\_\_\_\_\_\_\_\_\_\_

 c. Is this a property of all reflections? \_\_\_\_\_\_\_\_\_\_

3. Suppose $\overleftrightarrow{AB}$ || $\overleftrightarrow{CD}$. When lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AB}$ are mapped onto lines $\overleftrightarrow{A’B’}$ and $\overleftrightarrow{C’D’}$, $ \overleftrightarrow{A’B’}$ || $\overleftrightarrow{C’D’}$.

 a. Is this a property of all translations? \_\_\_\_\_\_\_\_\_

 b. Is this a property of all rotations? \_\_\_\_\_\_\_\_\_\_\_

 c. Is this a property of all reflections? \_\_\_\_\_\_\_\_\_\_

4. Suppose *M* is the midpoint of $\overbar{AB }$. When points *A*, *B*, and *M* are mapped onto points *A’*, *B’*, and *M’*, *M’* is the midpoint of $\overbar{A’B’}.$

 a. Is this a property of all translations? \_\_\_\_\_\_\_\_\_

 b. Is this a property of all rotations? \_\_\_\_\_\_\_\_\_\_\_

 c. Is this a property of all reflections? \_\_\_\_\_\_\_\_\_\_

**Part II.** The Geogebra file ctcoregeomACT1732 may be used for experiments in part II.

5. When is a line mapped onto itself?

 a. Suppose you have a translation by a vector from *X* to *Y*. Which lines are mapped onto themselves under this transformation?

 b. Suppose you have a 180° rotation about a point *P*? Which lines are mapped onto themselves under this transformation?

 c. Suppose you have a rotation about point *P*, but the angle of rotation is not a multiple of 180°. Are there any lines that are mapped onto themselves under this transformation?

 d. Suppose you have a reflection about line *l*. Which line is mapped onto itself under this transformation?

6. When is a point mapped onto itself?

 a. If you have a translation, are there any points that are mapped onto themselves? If so, which ones?

 b. If you have a rotation, are there any points that are mapped onto themselves? If so, which ones?

 c. If you have a reflection, are there any points that are mapped onto themselves? If so, which ones?

7. Which transformation or transformations will map a line *l* onto a line that is parallel to *l*?

 a. Does this work for any translations? If so, which ones?

 b. Does this work for any rotations? If so, which ones?

 c. Does this work for any reflections? If so, which ones?

**Part III.**

Summarize the properties you have discovered by filling in the blanks.

8. All isometries preserve distance and \_\_\_\_\_\_\_\_\_\_\_ measure. Pairs of parallel lines are mapped onto parallel lines. Midpoints of segments are mapped onto \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

9. Under a translation by the vector from *X* to *Y*, a line parallel to $\overleftrightarrow{XY}$ is mapped onto \_\_\_\_\_\_\_\_. A line that is not parallel to $\overleftrightarrow{XY}$ is mapped onto another line that is \_\_\_\_\_\_\_\_\_\_\_\_\_ to $\overleftrightarrow{XY}$

10. Under a rotation about a point *P*, the point *P* is mapped onto \_\_\_\_\_\_\_\_\_\_. A line through point *P* is mapped onto another line through \_\_\_\_\_\_\_\_\_. If the angle of rotation is 180° a line through *P* is mapped onto \_\_\_\_\_\_\_\_\_\_\_\_\_.

11. Under a reflection about line *l* every point on *l* is mapped onto \_\_\_\_\_\_\_\_\_\_\_. A line that is parallel to *l* is mapped onto another line that is \_\_\_\_\_\_\_\_\_\_\_\_ to *l*.