**Unit 2: Investigation 1 (3-6 Days)**

**Transforming Quadratic Functions**

**Common Core State Standards**

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

**Overview**

This Investigation builds both on students’ understanding of quadratic functions from Algebra 1 and on the transformations studied in Investigation 4 of Unit 1. Beginning with the parent function f(*x*) = *x*2, students will see how the transformations f(*x*) + k, f(*x* + k), kf(*x*), and f(k*x*) shift and stretch the graph, and how these transformations are related to writing quadratic functions in standard and vertex forms, which were introduced in Unit 8 of Algebra 1. Students are also introduced to some key ideas in calculus, such as concavity and end behavior, through investigating quadratic functions. These key ideas are revisited in the study of polynomial functions in Investigation 3. The number of days devoted to this Investigation will depend on the amount of material reviewed from Unit 8 of Algebra 1.

**Assessment Activities**

**Evidence of Success: What will students be able to do?**

* Be able to describe the graphs of *x*2 + k, (*x* + k)2, k*x*2, a*x*2 + b*x* + c, and a(*x* – h)2 + k, based on the values of k, a, b, c, and h.
* Given a quadratic function, be able to describe the features of the function (intercepts, intervals where it is positive and negative, intervals where it is increasing and decreasing, concavity, axis of symmetry, and end behavior).

**Assessment Strategies: How will they show what they know?**

* **Exit Slip 2.1.1** asks students to describe the graphs of quadratic functions written as *x*2 + k, (*x* + k)2, k*x*2, a*x*2 + b*x* + c, and a(*x* – h)2 + k by describing them as shifts or stretches to the graph of *x*2.
* **Exit Slip 2.1.2** asks students to take a particular quadratic function and to describe it according to the terms introduced in **Activity 2.1.5**.
* **Journal Prompt 1** asks students to identify the similarities and differences between linear and quadratic functions.
* **Activity 2.1.1a/2.1.1b Move It! Part Two** asks students to investigate functions of the form f(*x*) = *x*2 + k and g(*x*) = (*x* + k)2 for various values of k.
* **Activity 2.1.2 How to Move It! with Standard and Vertex Forms** asks students to describe the relationships between the transformations in **Activity 2.1.1a/2.1.1b** and the standard and vertex forms of quadratic functions.
* **Activity 2.1.3a/2.1.3b Stretch It! Part Two** asks students to investigate functions of the form f(*x*) = k*x*2 and g(*x*) = (k*x*)2 for various values of k.
* **Activity 2.1.4 How to Stretch It! with Standard and Vertex Forms** asks students to describe the relationship between the transformations in **Activity 2.1.3a/2.1.3b** and the standard and vertex forms of quadratic functions.
* **Activity 2.1.5 How Do Quadratic Functions Behave?** introduces students to a range of features of functions that we study, including *x*- and *y*-intercepts, increasing and decreasing, concave up and down, the axis of symmetry, and end behavior.

**Launch Notes**

Ask students to write down three different groups to which they belong. The groups can be anything—clubs at school, their immediate family, their church or synagogue, a group of friends on Facebook, etc. After they share, select one of the groups mentioned and point out what makes it different—for example, students working on the school yearbook all share a common purpose and goal of creating a lasting record of the students, teachers, and staff at the school this year. When we consider functions in mathematics, we usually call these groups “families of functions.” While the individuals in each family are different, they all share certain characteristics. In Algebra 1, we have already learned a great deal about the family of functions called linear functions. In Algebra 2, we will learn more about other families of functions, starting with Unit 2 on quadratic functions.

In order to begin this Investigation, students should have some “feel” for quadratic functions and to understand generally what a quadratic function is. Students will have already considered the growth of a quadratic function in **Activity 1.3.1** **Linear and Nonlinear Growth**, and how quadratic functions differ from both linear and exponential functions. However, this activity focuses on quadratic growth and does not include features of quadratic functions such as their graphs. For a class with little or no previous exposure to quadratic functions, some of the material from Algebra 1 makes an excellent starting point. If a class is familiar with a motion detector, **Activity 8.1.3 Rolling Ball CBR** from Algebra 1 is an interesting and exciting way to introduce quadratic functions. As an alternative, consider **Activity 8.1.1 Quadratics in the Kitchen** from Algebra 1, which uses a common mixing bowl to investigate the relationship between the radius of the bowl and the height of the water in the bowl.

**Teaching Strategies**

Note About Unit 8 in the Algebra 1 Curriculum: Students will have had differing levels of exposure to quadratic functions, and with differing amounts of intervening time, since having completed Algebra 1. Unit 8 from Algebra 1 covers some of the basic ideas of quadratic functions, and this material should be utilized as needed. If students have had little or no experience with quadratic functions, begin by reviewing some of the material from Investigation 1 of Unit 8 from Algebra 1 to familiarize students with quadratic functions. **Activity 8.2.5a Ball Bounce TI-84+** or **Activity 8.2.5b Ball Bounce TI-NSpire** from Algebra 1 is also recommended to provide students background on quadratic functions.

To complete Unit 2, it is important for students to understand the effects of the parameters a, b, and c when writing a quadratic function in standard form (f(*x*) = a*x*2 + b*x* + c), and the effects of the parameters h and k when writing a quadratic function in vertex form (f(*x*) = (*x* – h)2 + k). **Activities 8.1.6 Exploring the Parameters of *y* = a*x*2 + b*x* + c, Activity 8.2.2 Graphing Quadratic Functions in Vertex Form, 8.2.3 Exploring Parameters with Geometers Sketchpad, 8.2.4 Modeling with Quadratic Functions in Vertex Form,** or **8.2.6 Transforming Functions in Standard Form to Vertex Form** from Algebra 1 can provide this familiarity; these Activities should be reviewed and included in class as needed.

**Activity 2.1.1: Move It! Part Two** builds on **Activity 1.4.3** from Unit 1. There are two versions of this activity, one for the graphing calculator (**Activity 2.1.1.a**) and one for GeoGebra (**Activity 2.1.1.b**). To be consistent with the notation and terminology from **Activity 1.4.3**, be sure to use the language of an “outside change” (change to the dependent variable) with the transformation *x*2 + k and an “inside change” (change to the independent variable) with the transformation (*x* + k)2. Students should see that these transformations have the same effects as they did in **Activity 1.4.3**.

Note that if you use **Activity 2.1.1.b** (the Geogebra version), the slider changes the value of k from negative to positive. However, if you use **Activity 2.1.1.a** (the graphing calculator version), k is always positive but the operation sign changes from addition to subtraction. You may need to point out to the class that since a – b = a + (-b), a transformation may be described by f(*x*) = g(*x*) – b (where b is positive) or as g(*x*) + b (where b is negative).

**Group Activity**

**Activity 2.1.1a/2.1.1b Move It! Part Two** is designed for students to complete it in pairs.

**Activity 2.1.2 How to Move It! with Standard and Vertex Forms** should be completed after **Activity 2.1.1**, possibly as a homework assignment. This activity will connect students’ knowledge of transformations of the form f(*x*) + k and f(*x* + k) to the standard (f(*x*) = a*x*2 + b*x* + c) and vertex (f(*x*) = a(*x* – h)2 + k) forms of quadratic functions. Students should see that the parent function f(*x*) = *x*2 can be considered to be in either form (standard form with a = 1, b = 0, and c = 0; vertex form with a = 1, h = 0, k = 0). Students will see that transformations of the form f(*x*) + k are naturally considered to be in standard form, and that transformations of the form f(*x* + k) are naturally considered to be in vertex form.

**Differentiated Instruction (For learners needing more help)**

In order to complete **Activity 2.1.2**, students must be able to expand expressions such as (*x* + 3)2 to be able to write the transformation f(*x* + k) in standard form. **Activity 8.4.5** **Multiplying Polynomials** from Algebra 1 can be used to provide review and practice on this topic.

**Group Activity**

**Activity 2.1.3a/2.1.3b Stretch It! Part Two** is designed for students to complete it in pairs.

**Activity 2.1.3: Stretch It! Part Two** builds on **Activity 1.4.4** from Unit 1. This activity also has two versions, one for the graphing calculator (**Activity 2.1.3.a**) and one for GeoGebra (**Activity 2.1.3.b**). To be consistent with the notation and terminology from **Activity 1.4.4**, be sure to use the language of an “outside change” (change to the dependent variable) with the transformation k*x*2 and an “inside change” (change to the independent variable) with the transformation (k*x*)2. Students should see that these transformations have the same effects as they did in **Activity 1.4.4**.

Students should be cautioned that an inside change is different from an outside change in general, but sometimes they can result in the same graph. For example, the function

g(*x*) = 4*x*2 can be obtained from the parent function f(*x*) = *x*2 as the result of an inside change (g(x) = f(2x)) or an outside change (g(x) = 4f(x)). It is important that students know that with certain functions, the inside change f(k*x*) and the outside change kf(*x*) are dramatically different. Unit 6, where the graphs of trigonometric functions are examined, will demonstrate this difference.

**Activity 2.1.4: How to Stretch It! with Standard and Vertex Forms** should be completed once **Activity 2.1.3** is completed, possibly as a homework assignment. This Activity will connect students’ knowledge of transformations of the form f(k*x*) and kf(*x*) to the standard and vertex forms of quadratic functions. Students should see that either type of transformation (f(k*x*) or kf(*x*)) can naturally be considered to be in either standard or vertex form.

Upon completion of **Activity 2.1.4** you can use **Exit Slip 2.1.1**, which asks students to describe the graphs of *x*2 + k, (*x* + k)2, k*x*2, a*x*2 + b*x* + c, and a(*x* – h)2 + k.

To introduce **Activity 2.1.5: How do quadratic functions behave?**, you might show the following YouTube video, showing the beginning of the movie “Contact” (starring Jodie Foster and Matthew McConaughey): <https://www.youtube.com/watch?v=HL0waHkIkps>. The video starts with a shot of the Earth with the Sun in the background, and zooms out into outer space. (Note: the video is three minutes long; it is probably not necessary to show the entire three minutes). After the video, ask for students’ reactions and point out that the Earth and Sun look very different from far away than up close. We look at functions in a similar way in mathematics—sometimes it is useful to look at a function “up close” by zooming in on it, and at other times it is useful to “zoom out” and look at the “entire function” (like looking at the “entire universe”). Choose a quadratic function such as f(*x*) = (*x* – 1)2 – 1 to show on an overhead projector or a document camera with a graphing calculator, starting with a window [0,5] x [-5, 10]. This window shows only “one part” of the graph. Show how when you zoom in on a quadratic function, it “looks straight” (you can zoom in on any point to demonstrate this); but if you zoom out you see very different attributes of the function.

Students should be familiar with the following terminology: *x*-intercept of a function, *y*-intercept of a function, increasing function, decreasing function, and axis of symmetry. The following terms are likely new to students and should be explained: *concave up* (the function “bends upwards”), *concave down* (the function “bends downwards”), and *end behavior* of the function (as the independent variable increases or decreases without bound, what happens to the function). If you used **Activity 8.1.1** from Algebra 1 as a review, you might point out that the mixing bowl “held water” when it was upright, which is analogous to a quadratic function being concave up; if the bowl is turned upside down, it would “spill water” and be analogous to concave down. You might want to show the video, introduce this terminology using a few examples with the graphing calculator and overhead projector, and ask students to work through the first part of **Activity 2.1.5** in class, using the second part as homework.

**Differentiated Instruction (For learners needing more help)**

In order to complete **Activity 2.1.5**, students must be able to find the axis of symmetry and the coordinates of the vertex for a quadratic function written in either standard or vertex form. Activities from Algebra 1 that can be used as review for these topics include **Activity 8.1.6 Exploring the Parameters of *y* = a*x*2 + b*x* + c**, **8.2.2 Graphing Quadratic Functions in Vertex Form**, and **8.2.6 Transforming Functions in Standard Form to Vertex Form.**

After completing this activity, students can complete both **Exit Slip 2.1.2** and **Journal Prompt 1**.

**Journal** **Prompt 1**

Based on what you have learned about quadratic functions so far, find at least three ways that quadratic functions are similar to linear functions, and at least three ways that quadratic functions are different from linear functions. Be sure to include ideas such as concavity and end behavior that you learned in **Activity 2.1.5**.

[Possible similarities: both are polynomial functions, both functions have end behavior that tends to ∞ or -∞, both always have a *y*-intercept, the transformations considered in Unit 1 have the same effects on the graphs of both kinds of functions. Possible differences: the function growth is different, the shapes of the graphs of the functions are different, linear functions do not have a vertex or axis of symmetry, only quadratic functions can have even symmetry, linear functions are 1-1 but quadratic functions are not.]

**Closure Notes**

Consider using students’ responses from the journal prompt as a review of the similarities and differences of quadratic functions and linear functions.

Be sure to let students know that what they learn in Investigation 1 will continue to be an important part of their toolkit throughout Algebra 2 and beyond. They should see that transformations of quadratic functions in particular work in the same ways as the transformations they saw in Unit 1. It is not the *type* of function, but the *form* of the transformation, that determines the behavior of the function. In addition, the ways of looking at quadratic functions in **Activity 2.1.5** are used not only throughout Algebra 2, but also into Precalculus and Calculus. “Zooming In” and “Zooming Out” should become a familiar way of studying all new functions they encounter in the future.

**Vocabulary**

Absolute value

Axis of symmetry

Binomial expansion

Concave up/down

Decreasing function

End behavior

Family of functions

Increasing function

Infinity

Inside change

Outside change

Parabola

Parent function

Slope

Standard form of a quadratic function

Transformation

Vertex

Vertex form of a quadratic function

*x*-intercept

*y*-intercept

**Resources and Materials**

**All students should complete all activities in this Investigation.** **Activities 2.1.1a** and **2.1.3a** are written for the graphing calculator, and **Activities 2.1.1b** and **2.1.3b** are written using GeoGebra; you can choose which pair of these Activities to complete. Incorporate additional review from Unit 8 of Algebra 1 as needed and described above.

Activity 2.1.1a/2.1.1b Move It! Part Two

Activity 2.1.2 How to Move It! with Standard and Vertex Forms

Activity 2.1.3a/2.1.3b Stretch It! Part Two

Activity 2.1.4 How to Stretch It! with Standard and Vertex Forms

Activity 2.1.5 How Do Quadratic Functions Behave?

Graphing calculator/computer software with a graphing utility for all activities

Graph paper for all activities

Online access for Activity 2.1.1, Activity 2.1.3

Overhead projector or document camera or TI emulator or equivalent software

<https://www.youtube.com/watch?v=HL0waHkIkps>