**Activity 3.2.2 How Can Polynomial Long Division Be Helpful in Real-Life?**

People love to be entertained. Movies are a popular way to be entertained for a short period of time. Going to a movie theater is one way people access movies. The amount of money spent at movie theaters, in hundreds of millions, in the United States from 1990-1995 can be modeled by the polynomial function $M(t)=2t^{3}-11t^{2}+14t+27$ where *t* represents the number of years since 1990. The population of the United States, in tens of millions, can be estimated by the linear function $P(t)=2t+25$.

Advertisers are often interested in understanding how people spend their money. More importantly how they spend their entertainment money. It is important to moviemakers and advertisers to know, on average, how much people spend viewing movies. To find the per capita (average per person) spending, you can divide the money spent each year by the population of the US for that year. Complete the table below for the years 1990 to 1995.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **t** | $$M(t)=2t^{3}-11t^{2}+14t+27$$ | $$P(t)=2t+25$$ | $$C(t)=M(t)/P(t)$$ |
| 1990 | 0 | $$2(0)^{3}-11\left(0\right)^{2}+14\left(0\right)+27=27$$ | 2(0) +25 = 25 | 27/25=1.08 |
| 1991 | 1 |  |  |  |
| 1992 | 2 |  |  |  |
| 1993 |  |  |  |  |
| 1994 |  |  |  |  |
| 1995 |  |  |  |  |

What if we could divide polynomial *M(t)* by the linear function *P(t)*? Would the resulting polynomial be useful in computing the average spending by Americans at movie theaters?

What is the unit of measure of column C(t)?

First, let’s look at dividing integers and see if that procedure can lead us to a method for dividing polynomials.

Let’s use long division and divide 1,245 by 3. Remember that 3 is called the divisor, 1,245 is called the dividend and the answer is called the quotient. Also recall that sometimes division doesn’t produce a remainder equal to 0. In that case we write the remainder over the divisor after the quotient.

 a) 415 r 0 b) 414 r 3

 3|1245 3|1244

 12 -12

 04 04

 3 - 3

 15 14

 15 -12

 0 2

We can check the results of each problem above using multiplication as shown below:

a) 415 x 3 = 1245

b) 414 x 3 = 1242 and then by adding the remainder of 2 we get 1242 + 2 = 1244

*For more practice doing long division with integers see Geogebra worksheet ID 94350.*

Do the principles of long division apply when dividing polynomials? Let’s try.

Example 1: Use long division to find $(x^{2}+3x-40)÷(x-5)$.

 x + 8

x – 5 | x2 + 3x – 40

 (–) x2 – 5x Multiply the divisor by x since $\frac{x^{2}}{x}=x$

 8x – 40 Subtract. Bring down the next term

 (–) 8x – 40 Multiply the divisor by 8 since $\frac{8x}{x}=8$

 0 Subtract.

The quotient is x + 8. The remainder is 0. We can check our results using distribution as shown below.

 $\left(x+8\right)\*\left(x-5\right)=(x^{2}+3x-40)$

Example 2: Use long division to find $(a^{2}+3a-40)÷(3-a)$.

 –a – 10

–a + 3 | a2 + 7a – 11 For ease in dividing, rewrite 3 – a as –a + 3.

 (–) a2 – 3a Multiply the divisor by x since $\frac{a^{2}}{a}=a$

 10a – 11 Subtract. Bring down the next term

 (–) 10a – 30 Multiply the divisor by 10 since $\frac{10a}{a}=10$

 19 Subtract.

The quotient is –a – 10 with a remainder of 19. Write the solution as $-a-10+ \frac{19}{3-a}$.

In the space provided, check our results using distribution. Don’t forget to add the remainder after distributing!

*For more information about dividing polynomials, use the following Khan Academy video series:*

[*http://www.khanacademy.org/math/algebra2/polynomial\_and\_rational/dividing\_polynomials/v/polynomial-division*](http://www.khanacademy.org/math/algebra2/polynomial_and_rational/dividing_polynomials/v/polynomial-division)

*The results of dividing M(t) by P(t) are shown below. They can b created using the polynomial division app from* [*www.webgraphing.com/polydivision.jsp*](http://www.webgraphing.com/polydivision.jsp) *or* [*http://library.wolfram.com/webMathematica/Education/LongDivide.jsp*](http://library.wolfram.com/webMathematica/Education/LongDivide.jsp)*.*



Solution:

$$\frac{2x^{3}-11x^{2}+14x+27}{2x+25}= x^{2}-18x+232-\frac{5773}{2x+25}$$

Now that we know how to divide polynomials and we have the resulting polynomial from dividing M(t) by P(t), we can directly compute the per capita spending of Americans at movie theaters. Fill in the table below and compare the results to the first table.

|  |  |  |  |
| --- | --- | --- | --- |
| **Year** | **t** | $$C(t)=t^{2}-18t+232$$ | $$C\left(t\right)-\frac{5,773}{2t+25}$$ |
| 1990 | 0 | $$(0)^{2}-18\left(0\right)+232=232$$ | 232 –(5773/(2(0)+25)) = 1.08 |
| 1991 | 1 |  |  |
| 1992 | 2 |  |  |
| 1993 |  |  |  |
| 1994 |  |  |  |
| 1995 |  |  |  |

Practice Problems

Divide the following polynomials by the given divisor. If the remainder is not zero, write the remainder over the divisor

1. $(70m^{2}-4m-2)÷(7m+1)$
2. $(9r^{2}-27r+14)÷(3r-2)$
3. $(28a^{2}+13a-6)÷(7a-2)$
4. $(2r^{3}+r^{2}-46r-5)÷(r+5)$
5. $(a^{3}+10a^{2}+25a+6)÷(a+6)$
6. $\left(10a^{4}+82a^{3}-132a^{2}+148a-80\right)÷\left(10a-8\right)$
7. $ (20x^{2}+23x+8)÷(5x+2)$
8. $(x^{3}-12x^{2}+37x-14)÷(x-6)$
9. $(r^{3}+3r^{2}-10r+8)÷(r-1)$
10. $\left(6x^{4}-38x^{3}+16x^{2}-4x+51\right)÷\left(6x-8\right)$
11. $\left(v^{3}-2v^{2}\right)÷\left(v-2\right)$
12. $\left(x^{3}+5x^{2}+5\right)÷\left(x+5\right)$
13. $\left(r^{3}-7r+14\right)÷\left(r-1\right)$
14. $\left(3m^{3}+2m^{2}-5m\right)÷\left(m-1\right)$