**Activity 3.2.3 Polynomial Long Division and the Remainder Theorem**

The following set of problems will help you to discover a fascinating property of polynomials. Recall that dividing the polynomial, *P(x)* by *(x – a)* results in the quotient, q(x) and the remainder r(x) and the result can be written as *P(x) = Q(x)(x – a) + R(x).*

For each problem below, write the polynomial as the product of the quotient times the divisor plus the remainder.

1. Consider the polynomial function *p(x) = –3x2 + 5x + 4.*
2. Divide *p(x)* by *x – 3.* b. Evaluate *p(3).*
3. Consider the polynomial function *f(x) = 3x4 – 2x3 + 5x + 2.*
4. Divide *f(x)*  by *x – 4.* b. Evaluate *f(4).*
5. Consider the polynomial function $g\left(x\right)=x^{3}+4x^{2}-15x-18$.
6. Divide $g\left(x\right)$ by $x-3$. b. Evaluate g(3).
7. Consider the polynomial $p\left(x\right)=x^{2}+2x+1$
8. Divide *p(x)* by *(x + 1).* What is the remainder?
9. What is q(x)?
10. Does $p\left(x\right)= q\left(x\right)\left(x-a\right)+p(a)$?
11. Factor p(x). What do you notice about the factors of p(x)?

Can you make a conjecture about the relationship between dividing a polynomial by (x – a) and the results of evaluating p(a).

Conjecture:

What you should have discovered is that the remainder of dividing a polynomial by (x – a) is the same as evaluating the function, p(x) for x = a, or p(a) and that if p(a) evaluates to zero then (x – a) is a factor of p(x).

This can be shown to be true as follows:

Recall that $\frac{p(x)}{(x-a)}=q\left(x\right)+r(x)$ and it follows that $p\left(x\right)= q\left(x\right)\left(x-a\right)+r(x)$ where r(x) is equal to a constant, say r, since we’re dividing by the linear function (x - 1). Now let’s look at p(a). Since $p\left(x\right)= q\left(x\right)\left(x-a\right)+r $then $p\left(a\right)=q\left(a\right)\left(a-a\right)+r$. Since (a – a) is zero, then the product $q\left(a\right)\left(a-a\right)=0 $no matter what q(a) evaluates to leaving only r. This is called the Remainder Theorem, the remainder of $\frac{p(x)}{(x-a)}=p\left(a\right)$ or stated as

$p\left(x\right)= q\left(x\right)\left(x-a\right)+p(a)$.

The Factor Theorem states that given polynomial p(x), if p(a) = 0 for any real number **a** then (x – a) is a factor of p(x).

*For more information about dividing polynomials and the Factor and Remainder Theorem, use the following Khan Academy video series:*

<http://www.khanacademy.org/math/algebra2/polynomial_and_rational/polynomial-remainder-theorem-tutorial/v/polynomial-remainder-theorem>

1. Divide the polynomial $p\left(n\right)=-3n^{3}-4n^{2}+2n-7 by \left(n+2\right).$
2. Write the result in the form $q\left(x\right)\left(x-a\right)+r(x)$.
3. Is (n + 2) a factor of p(n)? Explain your answer.

Practice Problems

1. Use the Remainder Theorem to find the remainder of each of the following divisions.
2. $\left(n^{2}+5n+9\right)÷\left(n+3\right)$
3. $(r^{2}+2r+1)÷(r+1)$
4. $(m^{3}-7m^{2}+7m+6)÷(m-5)$
5. $\left(r^{4}+7r^{3}+7r^{2}-9r+27\right)÷\left(r+3\right)$

For problems 2 – 3, show that the value p(a) equals the remainder when p(x) is divided by (x–a) for the given values of x and for the given polynomial p(x).

1. Given polynomial p(x) = $x^{3}+10x^{2}+24x+12$.
	1. Divide p(x)by (x – 1) and write the result as q(x)(x – 1) + remainder.
	2. Find p(1).
2. Given polynomial p(x) = $x^{3}-5x^{2}+9x-17$.
	1. Divide p(x)by (x – 4) and write the result as q(x)(x – 4) + remainder.
	2. Find p(4).
3. Is the polynomial p(x) = $x^{3}-3x^{2}-16x+6$ divisible by (x + 3)? Show your work.
4. Is the polynomial p(x) = $-x^{5}+7x^{4}-12x^{3}+5x^{2}-21x+3$ divisible by (x - 4)? Show your work.
5. Is (x + 4) a factor of the polynomial p(x) = $x^{5}+8x^{4}+17x^{3}+8x^{2}+12x-17$? Show your work.
6. Is (x + 3) a factor of the polynomial p(x) = $x^{3}-x^{2}-13x-3$? Show your work.
7. Is (x - 3) a factor of the polynomial p(x) = $x^{50}-3x^{49}+3x-9$? Show your work.