**Why Does Exponential Growth Always Surpass Polynomial Growth?**

**Activity 3.6.3**

**One way to determine whether a function is growing faster is to compare the average rate of change over equal increments. Complete the questions to find out why exponential growth will always eventually surpass polynomial growth.**

1. Complete the table below for the given values of *x*.

|  |  |  |
| --- | --- | --- |
| *x* |  |  |
| 1 |  |  |
| 2 |  |  |
| 4 |  |  |
| 8 |  |  |
| 16 |  |  |
| 32 |  |  |
| 64 |  |  |
| 128 |  |  |

2. In the table above which function creates the larger numbers?

3. Find the average rate of change for each function between x=32 and x=16?

4. Interpret the meaning of the average rate of change between these two points.

5. As *x* increases, determine which graph is increasing faster and explain why?

**Another way to see that the rate of growth is greater for exponential functions with a base b>1 is to look at the growth factor of the function f(x), i.e., f(x+1)/f(x), when the input x is increased by 1.**

6. Complete the table below to examine the growth factor for the polynomial function g(x).

|  |  |  |  |
| --- | --- | --- | --- |
| *x* |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 4 |  |  |  |
| 8 |  |  |  |
| 16 |  |  |  |
| 32 |  |  |  |
| 64 |  |  |  |
| 128 |  |  |  |

7. What do you notice about the ratio ?

8. Complete the table below to examine the growth factor for the exponential function fx).

|  |  |  |  |
| --- | --- | --- | --- |
| *x* |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 4 |  |  |  |
| 8 |  |  |  |
| 16 |  |  |  |
| 32 |  |  |  |
| 64 |  |  |  |
| 128 |  |  |  |

9. What do you notice about the ratio ?

10. As *x* increases to infinity, compare the growth factors of the two functions.

11. What conjecture can you make about exponential functions with a base greater than 1 and polynomial functions based on your answer in #10?