**Activity 4.3.2 – Graphing Rational Functions II**

**Different Forms of Rational Functions**

In this activity you will learn that rational functions can be expressed in many forms. You will also examine *restrictions* to the domain of functions. When doing so, think about the values that cannot be in the domain. Remember that we cannot divide by zero.

1. In Y1 on your graphing calculator enter the function, . In Y2 enter the function . Set an appropriate window and graph the two functions.
2. What do you notice about the graphs?
3. Press 2nd GRAPH to look at the TABLE. What do you notice about the values shown in Y1 and Y2?
4. Why do you think this happened?
5. Let’s try two more functions. Enter into Y1 and into Y2.
	1. What do you notice about the graphs?
	2. Press 2nd GRAPH to look at the TABLE. What do you notice about the values shown in Y1 and Y2?
	3. Why do you think this happened?
	4. Show or explain how you can change $\frac{5x+1}{x}$ to $\frac{1}{x}+5$?
	5. If we started with , what would we need to do to add the two terms together? (Hint: Think about how to add fractions.)

As you can see,  and  are equal functions. Rational functions can be expressed in many forms just like linear and quadratic functions.

Let’s now consider the function $f\left(x\right)=\frac{x-3}{x-5}$.

The denominator is a binomial so this rational expression is a little more complicated than a rational expression with a monomial in the denominator. We could use transformations to graph this function, but it would be wise to explore other techniques that might be more efficient.

Consider the function’s domain and *x-* and *y*–intercepts. From your experience with graphing in **Activity 4.3.1**, you should recognize that this function has a vertical asymptote.

1. What is the equation of the vertical asymptote?
2. Complete the table of values below to examine the end behavior. You can use the table feature on your graphing calculator to help you. Think about your last activity and be on the lookout for behavior that points to the existence of a horizontal asymptote.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -100 | -10 | 0 |  | 5 | 6 | 10 | 100 | 10,000 |
| *y* |  |  |  |  | 0 |  |  |  |  |  |

1. Why do you have an error when *x* = 5?
2. As *x* approaches -10,000, what value is $f(x)$ approaching?
3. As *x* approaches 10,000, what value is $f(x)$ approaching?
4. What is the equation of the horizontal asymptote?
5. What is the *x*-intercept?
6. What is the *y*-intercept?
7. Using the table of values and your graphing calculator, graph the function $f\left(x\right)=\frac{x-3}{x-5}$ using the VA and the HA. Remember the VA and HA should be shown as dashed lines on your graph.



1. What is the domain of the function? Are there any restrictions? List them.

For **Questions 5 – 10**, your teacher will place you into a group and assign each group a function. Each group will determine the domain of the function. Let the domain assist you to locate any vertical asymptotes (VA). Each group will also determine the *x*- and *y*–intercepts and the end behavior to look for horizontal asymptotes (HA), and make a sketch of the graph of the function.

When your group is done, place your group sketch on the boardand be prepared to discuss your findings. Pay attention to the reports from the other groups. Rather than making a table of values for examining the end behavior to find HA, see if you can find a short cut for finding a HA that does not require creating a table of values.

1. Complete the table for the function,.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 | 0 |  | 2 | 5 | 10 | 100 | 10,000 | 10,000 |
| *y* |  |  |  | 0 |  |  |  |  |  |  |

1. Domain: List any restrictions of the domain.
2. *x*-intercept: *y*-intercept:
3. As *x* approaches -10,000, what value is $f(x)$ approaching?
4. As *x* approaches 10,000, what value is $f(x)$ approaching?
5. HA: VA:
6. How is the equation of the HA related to the leading coefficients for the function? (Hint: Think about division.)
7. Using your table of values and your graphing calculator, graph the function below. Remember to include the VA and HA.



1. Complete the table for the function, .

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 | -2 | -1 | 0 |  | 10 | 10,000 |
| *y* |  |  |  |  |  | 0 |  |  |

1. Domain: List any restrictions of the domain.
2. *x*-intercept: *y*-intercept:
3. As *x* approaches -10,000, what value is $f(x)$ approaching?
4. As *x* approaches 10,000, what value is $f(x)$ approaching?
5. HA: VA:
6. How is the equation of the HA related to the leading coefficients for the function? (Hint: Think about division.)
7. Use the table of values and your graphing calculator to graph the function below. Remember to include the VA and HA on your graph.



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 | -4 |  | 0 | 4 | 10 | 10,000 |
| *y* |  |  |  | 0 |  |  |  |  |

1. Complete the table for the function,
2. Domain: List any restrictions of the domain.
3. *x*-intercept: *y*-intercept:
4. As *x* approaches -10,000, what value is $f(x)$ approaching?
5. As *x* approaches 10,000, what value is $f(x)$ approaching?
6. HA: VA:
7. How is the equation of the HA related to the leading coefficients for the function? (Hint: Think about division.)
8. Use your table of values and graphing calculator to graph the function below. Remember to include the VA and HA.



1. Complete the table for the function,.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 |  | 0 | 2 | 3 | 10 | 10,000 |
| *y* |  |  | 0 |  |  |  |  |  |

1. Domain: List any restrictions of the domain.
2. *x*-intercept: *y*-intercept:
3. As *x* approaches -10,000, what value is $f(x)$ approaching?
4. As *x* approaches 10,000, what value is $f(x)$ approaching?
5. HA: VA:
6. How is the equation of the HA related to the leading coefficients for the function? (Hint: Think about division.)
7. Use your table of values and graphing calculator to graph the function below. Remember to include the VA and HA.



1. Complete the table for the function,.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 |  | -0.375 | 0 | 10 | 100 | 10,000 |
| *y* |  |  | 0 |  |  |  |  |  |

1. Domain: List any restrictions of the domain.
2. *x*-intercept: *y*-intercept:
3. As *x* approaches -10,000, what value is $f(x)$ approaching?
4. As *x* approaches 10,000, what value is $f(x)$ approaching?
5. HA: VA:
6. How is the equation of the HA related to the leading coefficients for the function? (Hint: Think about division.)
7. Use your table of values and graphing calculator to graph the function below. Remember to include the VA and HA.



1. Complete the table for the function,.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 |  | 0 | 2 | 3 | 10 | 100 | 10,000 |
| *y* |  |  | 0 |  |  |  |  |  |  |

1. Domain: List any restrictions of the domain.
2. *x*-intercept: *y*-intercept:
3. As *x* approaches -10,000, what value is $f(x)$ approaching?
4. As *x* approaches 10,000, what value is $f(x)$ approaching?
5. HA: VA:
6. How is the equation of the HA related to the leading coefficients for the function? (Hint: Think about division.)
7. Use your table of values and graphing calculator to graph the function below. Remember to include the VA and HA.



1. How do we find the *y*-intercept of a rational function? Is it similar to finding the *y*-intercept of a linear or a quadratic function?
2. How do we find the *x*-intercept of a rational function? Is it similar to finding the *x*-intercept of a linear function? Do you think there can be more than one *x*-intercept?

Notice that in each of these examples, both the numerator and denominator had the **same** degree.

1. When the exponents in the numerator and denominator are the same, what is the relationship between the leading coefficients and the HA? Did you find the shortcut? Check with your teacher to see if your shortcut is correct.
2. Now that you have worked with horizontal and vertical asymptotes, try to generate a class definition of these special lines.

Now let us examine rational functions that have a **quadratic** in the denominator.

1. Consider the function. Complete the table below.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 | -2 | -1 | 0 | 1 | 2 | 10 | 10,000 |
| *y* |  |  |  |  |  |  |  |  |  |

1. Domain:

List any restrictions of the domain.

1. *x*-intercept: *y*-intercept:
2. As *x* approaches -10,000, what value is $f(x)$ approaching?
3. As *x* approaches 10,000, what value is $f(x)$ approaching?
4. HA: VA:
5. Use your table of values and graphing calculator to graph the function below. Remember to include the VA and HA.



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 | -3 | -1 | 0 | 3 | 10 | 10,000 |
| *y* |  |  |  |  |  |  |  |  |

1. Consider the function, . Complete the table below.
2. Domain:

List any restrictions of the domain.

1. *x*-intercept: *y*-intercept:
2. As *x* approaches -10,000, what value is $f(x)$ approaching?
3. As *x* approaches 10,000, what value is $f(x)$ approaching?
4. HA: VA:
5. Use your table of values and graphing calculator to graph the function below. Remember to include the VA and HA.



1. Consider the function,. Complete the table below.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -10,000 | -10 | -3 | -2 | 0 | 1 | 10 | 10,000 |
| *y* |  |  |  |  |  |  |  |  |

1. Domain:

List any restrictions of the domain.

1. *x*-intercept: *y*-intercept:
2. As *x* approaches -10,000, what value is $f(x)$ approaching?
3. As *x* approaches 10,000, what value is $f(x)$ approaching?
4. HA: VA:
5. Use your table of values and graphing calculator to graph the function, below. Remember to include the VA and HA.



Notice how in each of the functions in questions 15 – 17, the degree of the denominator was larger than the degree of the numerator.

1. What was the HA in each of these examples?
2. How is the VA related to the restrictions of the domain?

Let’s summarize what we discovered so far about the HA and VA for rational functions.

1. When the degree of the numerator and denominator are the same, how do we find the value of the HA?
2. When the degree of the denominator is larger than the degree of the numerator, what will the HA always be?
3. Fill in the blanks:

The VA is related to the restrictions of the domain and can be found by setting the

denominator of the function equal to \_\_\_\_\_\_\_\_\_ and solving for the variable. If the

denominator is a quadratic degree or higher degree, then you will need to \_\_\_\_\_\_\_\_\_\_\_\_\_\_

in order to find a VA.

1. If your class definition made reference to just one vertical asymptote you may need to edit it now.
2. In **Activity 4.3.1** you were asked if the graph of a rational function could cross a vertical asymptote. We will explore this idea more in a later activity. Based on your experience so far, can it? Explain. Can it cross a horizontal asymptote? Explain.