**Unit 3: Investigation 5 (3-4 days)**

**Applications of Polynomials**

**Common Core State Standards**

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**Overview**

Investigation 5 examines the application of polynomial functions and expressions in solving problems. One investigation is designed to use students’ knowledge of polynomial functions to solve problems that can be modeled by polynomial functions. Students will answer questions related to properties of the graph of the function such as x-intercepts, y-intercepts, intervals where the function is increasing or decreasing, and relative maxima and minima of the function. Activity 3.5.3 is an extension of the Binomial Theorem in which polynomials are used as generating functions in order to solve combinatorics problems. This activity is intended for advanced students going into STEM areas.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Create a polynomial function model from real-life data using higher order polynomials using regression features of a graphing utility.
* Interpret the meaning of characteristics of a polynomial function in the context of a real-world problem.
* Interpret the meaning of the coefficients of a polynomial that is the result of the product in a generating function.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 3.5.1** asks students to model a second situation in which a polynomial function is needed to model the data.
* **Journal Prompt 1** asks the students to identify the features of the polynomial that help the students decide what degree the polynomial model should be.
* **Journal Prompt 2** will ask the students which method of deriving the equation, using knowledge of the x-intercepts and number of changes in the curve or using the regression features of a graphing utility are more effective in modeling the roller coaster ride.
* **Journal Prompt 3** will ask students to create their own example of a question that generating functions can be used to answer.
* **Performance Task:** As a potential performance task for the unit, students would be asked to find data for a relationship that could be modeled by a polynomial function. The students would create the model and pose questions that can be answered based on the model. The teacher can suggest possible research topics, but if possible, students should choose the topic to be researched.
* **Activity 3.5.1** **Technology Companies - Can You Count on Them?**

will have students create a scatter plot for provided data and the students will make conjectures as to what polynomial function would best fit the data and why.

* **Activity 3.5.2** **Roller Coaster Redux** will have students watch a youtube video that shows two young children who try to fit a curve to model the shape of a desired roller coaster. The children are not successful. Students can discuss what the children did wrong and try to correct it based on their knowledge of x-intercepts and modeling of a polynomial.
* **Activity 3.5.3(**+)**Can You Count It?** will investigate another application of polynomials in the field of combinatorics called generating functions. This is an optional activity to be used time permitting.

**Launch Notes**

Students will consider a problem in the business world where profit or revenue needs to be modeled in order to make financial decisions. For the first activity, students are asked to create a revenue model for Electronic Arts, Inc. (EA Inc.), the company that develops software for video games such as NFL Madden and FIFA Soccer. Students will need to determine what degree polynomial they think best fits the data, justifying their choice of function.

**Teaching Strategies**

**Activity 3.5.1** will launch the investigation with the following problem. The revenue for the software company from 1998 to 2004 is listed in the table below. After creating the scatter plot for the data, students will conjecture about what polynomial function would best fit the data. (Data Source: Electronic Arts, Inc., Annual Reports, March 31, 2002 and 2004, retrieved from <http://dufu.math.ncu.edu.tw/calculus/calculus_pre/node7.html>, January 6, 2015).

|  |  |
| --- | --- |
| Years Since the End of Fiscal Year 1998 | Net Revenues in Billions of Dollars |
| 0 | .909 |
| 1 | 1.222 |
| 2 | 1.420 |
| 3 | 1.322 |
| 4 | 1.725 |
| 5 | 2.482 |

The cubic regression on the data would yield the cubic function: *f(x) = .0439x3 – .275x2 + .595x + .900.* Questions in the activity will include questions such about the end behavior of the function, the x-intercepts of the function, whether the function will be accurate for the future. Data for the revenue of the company for more recent years can be researched and students can discuss whether the cubic model is a good fit for the data. Follow-up scenarios will determine if the students are able to create and interpret accurate models for given data.

As an assessment for this activity, **Exit Slip 3.5.1** will ask students to create another model for data that gives the revenues from sales of oysters in the Long Island Sound for the period from 1999 to 2008. This Exit Slip could flip-flop with the data in Activity 3.5.1 if a teachers feels that data from closer to Connecticut would be of greater interest to the students. Additional data can come from other data sources if deemed more relevant.

**Group Activity**:

Group students to create the scatter plot and the polynomial function that they think best fits the data.

**Differentiation**:

This lesson can be scaffolded to provide more detailed instructions to the students. For example, if the teacher feels that the students are not ready to decide what the degree of the polynomial should be, then the teacher can instruct students to use a particular degree for their model.

**Journal Prompt 1** asks the students to identify the features of the polynomial that helps the student decide what degree the polynomial model should be.

**Activity 3.5.2** will revisit the roller coaster problem. To introduce the activity, students will watch a youtube video that shows two young children who try to fit a curve to model the shape of a desired roller coaster. The children are not successful and the video ends with the children asking why. Students can discuss what the children did wrong and try to correct it based on their knowledge of x-intercepts and modeling of a polynomial. The url for the youtube video is: <https://www.youtube.com/watch?v=RcfMdHD_4ZI>.

As a follow-up to this introduction, students will be asked to find the image of a roller coaster and create a polynomial function to model the shape of the roller coaster. If students can’t find an appropriate image, they can use the image below as found on the internet at Mrs. Mazzott’a Roller Coaster Project url: <https://sites.google.com/site/mrsmazzottassciencepage/home/energy/roller-coaster-project> The image given in her project is shown below, but students are encouraged to find their own image if possible. In attempting to fit a curve to the roller coaster, students will need to make some assumptions about the placement of the x- and y-axes, the number of x-intercepts, and the degree of the polynomial.

After coming up with the equation using properties of the polynomial, students can insert the image of the roller coaster into a coordinate system, create order pairs that model key points on the curve and use the regression features of a graphing utility like a TI-84 or GeoGebra to find an equation that fits the curve.



**Group Activity**:

Pair students as appropriate to figure out what went wrong with the children’s function to approximate their desired roller coaster. Students will continue to work in pairs as they complete the remainder of the investigation.

**Differentiation**:

The teacher can determine whether the graph of the roller coaster should be presented to the students with the x- and y-axes already shown or whether to have the students make that decision themselves.

**Journal Prompt 2** will ask the students which method of deriving the equation, using knowledge of the x-intercepts and end behavior of the curve or using the regression features of a graphing utility, will be more effective in modeling the roller coaster ride.

**Activity 3.5.3** (+)will investigate another application of polynomials in the field of combinatorics called generating functions. This activity is also intended for advanced students preparing for STEM careers. A generating function is a polynomial expression whose coefficients can be used to solve counting problems. Consider the problem of the outcome of a five game series in sports. The result of the expansion of the binomial *(x+1)5 = 1•x5 + 5•x4 + 10•x3 + 10•x2 +5•x1 +1•x0* can be used to determine the number of times a team would win 5 games, 4 games, • • •, 1 game, or 0 games. Although students have already studied the Binomial Theorem, they would benefit from visualizing how to get the individual terms in the product of *(x + 1)5*. For example, the way to get two wins would be illustrated in the following manner:

$\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)$

$\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)$

$\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)$

$\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)$

•

•

•

$\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)•\left(\begin{array}{c}x\\+\\1\end{array}\right)$

The images indicate each way to get a term of x2 in the product *(x+1)•(x+1)•(x+1)•(x+1)•(x+1)*. Two of the five binomial factors contribute an x to the product and the other three binomial factors contribute a 1 to the product. The 10 ways for that to happen parallel the 10 subsets of 2 elements from a set of 5 elements.

Students would then consider the problem of the number of ways that 6 women could be interviewed from a group of 20 women where there were 6 white females, 7 African American females, 4 Hispanic American females, and 3 Asian American females and at least one representative from each group needs to be taken. Note that it does not matter which members of the group are chosen. In the mathematics sense we would say that the members of the subgroups were indistinguishable. Each group respectively could be represented by the following generating polynomials: *(x1 + x2 + x3 + x4 + x5 + x6); (x1 + x2 + x3 + x4 + x5 + x6 +x7); (x1 + x2 + x3 + x4); and (x1 + x2 + x3).* When the product of these polynomials is calculated, the coefficient of the term with the exponent 6 would give the number of ways the 6 people could be chosen. For example, when the term *x3* in the first polynomial is multiplied by *x1* in each of the remaining polynomials, the result is the term *x6*. This term represents the possibility of interviewing 3 white females and 1 each of the other three groups. In order to make the computation easier, a Computer Algebra System (CAS) as found in GeoGebra would be used to compute that coefficient. The resulting product is:

*x4 + 4x5 + 10x6 + 19x7 + 30x8 + 42x9 + 53x10 + 61x11 + 64x12 + 61x13 + 53x14 + 42x15 + 30x16 + 19x17 + 10x18 + 4x19 + x20*

Note that the coefficient of the term *x6* is 10, the correct number of possible interview groupings with 6 people. The worksheet for this application would include questions that restrict the number of people that you must have as interviewees. For example, you could say that there must be as least two women from each group and you would now interview 10 of the women. How would this impact the problem? Or you could put a restriction on one group, such as at least three women who are African American or exactly two Hispanic American women.

A second scenario that uses generating functions to solve a combinatorics question is finding possible sums for three coins, where each coin has to be either a penny, a dime, or a quarter. The generating polynomial representing this situation is *(x1 + x10 + x25),* where *x1* is a penny, *x10* is a dime, and *x25* is a quarter. Then the product *(x1 + x10 + x25)3* would represent the possible ways that the three coins could be chosen with the exponent being the value of the three coins. Again using a CAS, the product is:

*(x1 + x10 + x25)3 = x3 + 3x12 + 3x21 + 3x27 + x30 + 6x36 + 3x45 + 3x51 + 3x60 + x75.*

Students could then answer questions such as what happens if you have 10 coins. What amount of change can you make in the most different ways. Which amounts of change have the same number of ways of being made and why would this be true?

For closure in this lesson, students would be asked to create their own example of a question that generating functions can be used to answer.

**Group Activity**:

Place the students in pairs of approximately equal ability to work with the Computer Algebra System (CAS) in calculating and interpreting the coefficients of the generating functions .

**Differentiation**:

This lesson is generally an enrichment application of polynomial multiplication but can be modified to make it accessible to groups needing assistance by the use of manipulatives. For example, the problem with choosing members of the different groups to be interviewed could be modeled with different colored chips or different coins (penny, nickel, dime, quarter). The coins could be used for a problem about making change with those coins. The problems could also be modified to start with even simpler problems that would be more accessible to students needing assistance. In general, this would be an enrichment topic for more advanced students.

**Journal Prompt 3** will ask students to create their own example of a question that generating functions can be used to answer.

**Closure Notes**

Closure for these lessons will require students to make generalizations about when a polynomial function would be the best model for certain data. Students should recognize that data whose rate of change varies from increasing to decreasing for different intervals of data would not be modeled by linear, exponential, or power functions. Therefore, polynomial, rational, or trigonometric functions would be more appropriate models for the data.

**Vocabulary**

generating polynomial

generating function

mathematical model

regression equations

scatter plots

**Resources and Materials**

**Activities 3.5.1 and 3.5.2 should be done by all students. Activity 3.5.3 should only be considered if time permits and is intended for Stem intending.**

Activity 3.5.1Technology Companies - Can You Count on Them?

Activity 3.5.2 Roller Coaster Redux

Activity 3.5.3(+)Can You Count It?

Data Source: Electronic Arts, Inc., Annual Reports, March 31, 2002 and 2004, retrieved from <http://dufu.math.ncu.edu.tw/calculus/calculus_pre/node7.html>, January 6, 2015

Evered, Lisa J. and Brian Schroeder. 1991. “Counting with Generating Functions.” In *Discrete mathematics across the Curriculum: K-12.* 1991 Yearbook of the National Council of Teachers of Mathematics, edited by Margaret J. Kenney and Christian R. Hirsch. Reston, VA: NCTM, 1991, 143-148.

Mrs. Mazzott’a Roller Coaster Project url: <https://sites.google.com/site/mrsmazzottassciencepage/home/energy/roller-coaster-project>

Kahan, Jeremy A. and Terrence R. Wyberg. 2003. “Technology Matters: An Invitation to Generating Functions with CAS.” In *Computer Algebra Systems in Secondary School Mathematics Education,* edited by James T. Fey. Reston, VA: NCTM, 151-162

CAS in GeoGebra or internet applet.