**Unit 4: Investigation 5 (3 Days)**

**Rational Equations**

**Common Core State Standards**

A.SSE.1b Interpret complicated expressions by viewing one or more parts as a single entity..

A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous roots may arise.

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x); find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

**Overview**

In this investigation students will solve equations that contain at least one polynomial expression with a variable in the denominator. Because finding the solution involves multiplication of both sides of the equation by an expression involving the variable, it is possible that the resulting equation is not equivalent to the original. Students will revisit the Multiplication Principle for Solving Equations that states you can multiply both sides of an equation by the same *nonzero* number or expression and still have an equivalent equation and recognize there is now the possibility they may be multiplying by zero since they need to multiply by an expression and not just a number.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Interpret and solve an application problem that can be modeled by a rational or power function.
* Verify that a solution makes sense in the context of the problem.
* Check proposed solutions to be sure no root is an extraneous one.
* Translate a verbal description of a relationship into an equation.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit slip 4.5** Students will solve rational equations and one has an extraneous root.
* **Journal Prompt 1** asks students to explain how it is possible to obtain an answer and yet not have it be a solution of the original equation.
* **Journal Prompt 2** asks students to respond to: “Although it is wise to check every potential solution to an equation, you have seen it is very important for a rational equation because there might be extraneous roots. But if you are pressed for time, you can short cut the checking process by instead just checking what? Explain.”
* **Activity Sheet 4.5.1 More Equations and Problems.** Students working in pairs will move around the room from station to station provided they have solved their rational equation correctly.
* **Activity Sheet 4.5.2 Solving Equations with Fractions and Rational Expressions**. Students will solve rational equations, some in context.
* **Activity Sheet 4.5.3 More Equations and Problems.** Students will solve a variety of verbal situations as well as ordinary equations. The equations will be linear, quadratic, polynomial, square root, rational and equations with fractions.

**Launch Notes**

You could show the following movie clip from the movie, Little Big League available from Math in the Movies at <http://www.math.harvard.edu/~knill/mathmovies/m4v/league.webm>. The need to be able to solve word problems and to use fractions and rational expressions to express quantities involving fractions should be evident as they watch the clip. Students should then solve this problem that involves fractions.

Next have students consider solving the equation 1/(x – 4) – 5/(x + 4) = 8/x2 – 16). They should first be asked to consider the domains of the rational expressions within the equation and note that neither 4 nor - 4 can be used. Groups or pairs of students should attempt to solve the equation. Ideally at least two methods of solving should be discovered. Either multiply both sides by (x – 4) ( x + 4) or others may suggest adding the two expressions on the left and then using the LCD. Still others may argue that since both fractions are equal and both denominators are the same that then both numerators must be equal. Students should be encouraged to explain why their method works. But regardless of the method used this problem has no solution. Four makes a denominator zero so it cannot be a root. Why aren’t all the equations in their solving process equivalent? For those that combined the expressions on the left side of the equation and then used the logic that since the fractions are now equal and the denominators are equal so they can just use the numerators, they inadvertently threw away the restrictions. The new equation has a solution but the original one does not because it came with a restricted domain. For those that multiplied each side by the LCD, they multiplied by an expression that was zero so the old equation has no solution, the new one does because a root was introduced.

**Teaching Strategies**

**Activity sheet 4.5.1** can be distributed to pairs of students. Each pair can solve their equation and then look for the solution on pieces of paper taped around the room . The solution will then direct them to the next problem they need to solve and so on. If none of the posted solutions matches a pair’s solution they need to try again and if necessary go to the teacher post to obtain a hint. A teachers’ resource sheet with one equation per page is provided. If there is time each pair can present a solution and **Activity Sheet 4.5.2** can be assigned for homework in part or in whole. Otherwise students can be handed a copy of all the problems and students can solve the ones for homework that they did not complete in class from **Activity Sheet 4.5.1.**

 **Activity Sheet 4.5.1** supports a group activity and contains rational equations to solve in class. A few have no solution.

**Differentiated Instruction (For Learners Needing More Help).** Start students with problems that have fractions in them but keep the variable in the numerator Then move to rational equations that have the variable in the denominator.

Once all the problems have been completed in **Activity 4.1.1** each pair should present a solution. Students can be asked to solve some problems like, “The numerator of a fraction is 2 less than its denominator. The sum of the fraction and its reciprocal is 130/63. Find the numerator and the denominator if each is a positive integer.” Teacher note: 7/ 9 is the fraction and students need to solve a quadratic. Or a problem such as Jose is taking a two period test that has 65 questions that all count equal weight. When the teacher gives him part two on day 2 she tells him he got all 38 of yesterday’s questions correct. How many more questions must he answer correctly to get a 94 percent on the test? Students can also use their new skill to find out if the graph of a rational function will cross its horizontal asymptote. By the end of the second day students should be able to respond to the **Journal Prompt 1** and should have some or all of **Activity Sheet 4.5.2** assigned**.** The checks for # 2 and 3 are challenging because the roots are fractions themselves and # 8 has a fraction for *a* so that determining the length and width provides a workout for students.

**Journal Prompt 1** Explain how it is possible to obtain an answer and yet not have it be a solution of the original equation. Students might respond that when the Multiplication Rule for Solving Equations is used, it only guarantees equivalent equations when you multiply by a nonzero number. When you multiply by an expression with a variable there is the chance of multiplying by zero, that is, of multiplying by one of the values excluded from the domain of the expressions in the equation. The equations may not be equivalent and the new equation may have a root or roots the original did not.

**Activity sheet 4.5.3** is a mixture of applications and ordinary skill equations that were studied throughout the unit so direct, indirect (inverse) variation and rational equations are represented as well as a few linear, square root, quadratic, cubic equations from the first three units. The class could be split into 2 teams and the problems used as contest questions. **Exit slip 4.5** could be distributed.

**Differentiated Instruction (Enrichment).** Have students make up an application problem that needs a rational equation and provide its solution. Have students make up an equation that has extraneous root(s).

**Journal prompt 2** could be distributed.

**Journal Prompt 2** Although it is wise to check every potential solution to an equation, you have seen it is very important for a rational equation because there might be extraneous root. But if you are pressed for time , you can short cut the checking process by instead checking what? Students might say that even though you may have made an arithmetic error and only doing a check will catch that, you can at least check for extraneous roots by just making sure the value does not make a denominator zero in the original equation.

**Closure Notes**

Have the class summarize a strategyfor solving rational equations. How is solving them like other equations they have solved this year? How is it different?

**Vocabulary**

Average cost

Area

Difference

Domain of an expression

Extraneous solution

Like and unlike denominators

Perimeter

Product

Quotient

Sum

Surface area

**Resources and Materials**

**All activities 4.5.1- 4.5.3 should be completed.**

Activity Sheet 4.5.1 More Equations and Problems

Activity Sheet 4.5.2 Solving Equations with Fractions and Rational Expressions

Activity Sheet 4.5.3 More Equations and Problems

Graphers

Movies clip <http://www.math.harvard.edu/~knill/mathmovies/m4v/league.webm>.

Cards to post on the wall for activity, colored Xerox paper (optional) 4.5.1

[www.physicsclassroom.com](http://www.physicsclassroom.com) for the Doppler effect