**Unit 4: Investigation 5 (3 Days)**

**Similarity in Right Triangles**

**Common Core State Standards**

* G-SRT.B.4 Prove theorems about triangles. Theorems include: *a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
* G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
* G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.\*

**Overview**

In this investigation students will begin by constructing an altitude to the hypotenuse of a right triangle. This will create two new triangles that are similar to each other and the original triangle. Students will then use this fact to prove the Pythagorean theorem.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Decompose figures and prove that triangles that make up the whole are similar.
* Prove the Right Triangle Similarity Theorem and use it to prove the Pythagorean Theorem.
* Solve problems involving proportions in right triangles.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 4.5.1** requires students to demonstrate understanding of the Right Triangle Similarity Conjecture
* **Exit Slip 4.5.2** requires students to apply the Pythagorean Theorem to solve a real-world problem.
* **Journal Entry** has students explain the connection between similar triangles and the Pythagorean Theorem.

**Launch Notes**

You may begin by giving each student a notecard or piece of paper. Have them cut the paper along one of its diagonals to create two congruent triangles. Then take one of the two triangles and draw an altitude from the vertex at the right angle to the hypotenuse. Then have students cut along this altitude. Next ask students what relationship exists among the three triangles and how they know this. Coach the students into figuring out for themselves that the three triangles are all similar to each other.

**Teaching Strategies**

In **Activity 4.5.1** **Right Triangles** students will discover that an altitude to the hypotenuse of a right triangle from the opposite vertex will create two new triangles that are similar to each other and the original triangle. At first students will see this through a hands-on activity using a notecard or piece of paper. To learn if it is true for other right triangles, students will use dynamic geometry software to check many other examples. When students are ready they will write a Right Triangle Similarity Conjecture based on the findings of the whole class. (If student computer access is not available, the teacher may do the second half of the activity as a demonstration.)

**Group Activity**

Activity 4.5.1 involves making a conjecture based on measurement. Encourage students to work in groups and then compare results among the groups to determine if their evidence supports the Right Triangle Similarity Conjecture.

**Activity 4.5.2** **Proving the** **Right Triangle Similarity Theorem** provides a proof of the conjecture that the altitude to the hypotenuse of a right triangle from the opposite vertex forms two new triangles that are similar to each other and the original triangle. There are two versions of this activity. **Activity 4.5.2a** is open ended, whereas **Activity 4.5.2b** has students filling in blanks.

**Differentiated Instruction (For Learners Needing More Help)**

Give Activity 4.5.2b and Activity 4.5.4b to students who are not yet ready to write complete proofs on their own.

**Exit Slip 4.5.1** may be given after **Activity 4.5.2** is completed.

**Activity 4.5.3** **Geometric Means** may be skipped if there is no time, but useful for more advanced students. Students continue to explorer the altitude drawn to the hypotenuse of a right triangle. The length of the altitude to the hypotenuse is the geometric mean between segments on the hypotenuse; each leg is the geometric mean between hypotenuse and its projection onto the hypotenuse. Both of these results follow from the Right Triangle Similarity Theorem.

**Differentiated Instruction (Enrichment)**

An extension of this activity compares and contrasts the geometric mean with the arithmetic mean. A well-known theorem states that the arithmetic mean is greater than or equal to the geometric mean. For an algebraic proof see http://en.wikipedia.org/wiki/Inequality\_of\_arithmetic\_and\_geometric\_means.

**Activity 4.5.4** **Using Similar Triangles to Prove the Pythagorean Theorem** presents a classic proof of theorem, which students proved informally in Unit 1. Again there are two versions of this activity. **Activity 4.5.4a** is open ended, whereas **Activity 4.5.2b** has students filling in blanks.

Students may use this theorem to solve application problems involving the right triangle. It should be noted that while students have used and solved simple problems involving the Pythagorean theorem, these problems require several steps. Students are then asked to create their own application problem using the Pythagorean Theorem.

**Differentiated Instruction (Enrichment)**

There are over 300 known proofs of the Pythagorean Theorem. Some student may want to do research to find some of the others.

**Journal Entry** Explain how similar triangles are used to prove the Pythagorean Theorem. Then try to recall other proofs of the theorem you may have seen. Compare and contrast the different proofs of this theorem. Look for students to reference at least one other proof of the theorem.

**Exit Slip 4.5.2** may be given after **Activity 4.5.2** is completed.

In **Activity 4.5.5** **Right Acute or Obtuse?** students will explore side lengths of triangles that form right, acute, or obtuse triangles. This will lead them to making a conjecture about the converse of the Pythagorean Theorem. For this activity each group will need a pieces of string about 50 cm in length, and two paper clips.

In **Activity 4.5.6 The Converse of the Pythagorean Theorem** the proof of the converse is based on Euclid’s proof in his Book I Proposition 48. Students will then use this result to solve problems involving the converse.

**Closure Notes**

Have students share the problems that they developed in Activity 4.5.4.

**Vocabulary**

Geometric mean

**Theorems**

**Right Triangle Similarity Theorem** If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

**Pythagorean Theorem** In a right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

**Converse of Pythagorean Theorem** If *a*2 + *b*2 = *c*2, where *a*, *b*, and *c* are the lengths of the sides of a triangle, then the triangle is a right triangle with *c* as the length of the hypotenuse.

**Resources and Materials**

Activities:

 Activity 4.5.1 Right Triangles

 Activity 4.5.2 The Right Triangle Similarity Theorem

 Activity 4.5.3 Geometric Means (may be skipped)

 Activity 4.5.4 Similar Triangles and the Pythagorean Theorem

 Activity 4.5.5 Right, Acute, Obtuse?

 Activity 4.5.6 The Converse of the Pythagorean Theorem

Videos:

<http://www.projectmathematics.com/pythag.htm> has an extensive collection of videos on the Pythagorean Theorem.

Euclid’s proof of the converse of the Pythagorean Theorem is found in Book I Prop. 48

<http://aleph0.clarku.edu/~djoyce/java/elements/bookI/propI48.html>

String and paper clips for Activity 4.5.5.