**Activity 5.2.4 Equations Involving Logarithms**

You have already solved some logarithmic equations by rewriting them in exponential form. Now try a few more that may make you think even harder. Provide an exact solution where possible and an approximation to three decimal places. Two new tools to add to your equation solving tool box are stated below. They are consequences of the fact that the logarithmic family and exponential family are one-to-one functions.

**If A, B and b are positive real numbers and b ≠ 1 the logbA = logb B if and only if A = B.**

**If a > 0 and a ≠ 1, then ax = ay if and only if x = y.**

Solve:

1. log x = 4.5. So the exact solution is \_\_\_\_ and an approximation to 2 decimal places is \_\_\_\_\_
2. logx 5 = 2. You met this quadratic equation in unit 2. Be careful when you write the solution. \_\_\_\_\_\_\_\_\_\_\_There may be an extraneous root. Why?\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. ln x = 2 so \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. log4 x = log412. Use your new logarithm of both sides theorem and get \_\_\_\_\_
5. 5 + log 3 = log(2x + 4). Hint: subtract log 3 from both sides and use one of your log rules.
6. ln 3x2 = ln x + ln 7
7. (1/3)log x – 2 = log 1000
8. ln *e*4 + 5 ln x = 9
9. 4 log x = 10log 6 – log x
10. ln 12 + ln x = 8
11. The world population reached 6 billion on October 12, 1999. If we assume a growth rate of 1.4% when will the population reach 7 billion? Find out when the world population did reach 7 billion. How good was your prediction? Hint: This is a continuous compounding situation.
12. Carbon - 14 which is used for archaeological dating, has a half-life of 5730 years. What is its decay rate? \_ \_\_\_\_\_\_\_\_ (Hint: If you have 50 mg present now, then in 5,730 years how much will be present? Use this information and the fact that we need a continuous growth/decay model to find r.)
13. Three finely made ancient spears were found in a coal mine excavation near Hanover, Germany in 1997. Until this discovery was made it had been thought that humans began hunting about 40,000 years ago. These 6 to 7.5 foot spears were used to hunt horses, elephants and deer in the area. Only 9.676 X 10-20 percent of the carbon-14 remained in the spears. When were the hunters using them to hunt horses ? \_\_\_\_\_\_

Note: in 1911 a spear was found in England that dates more the 40,000 years ago but it was thought this was an isolated find. The discovery in Schoningen, Germany in 1997 provided evidence that early ancestors were hunting much earlier than 40000 years ago and the theory has now been changed to \_\_\_\_\_\_\_\_\_\_\_(you fill in the number of years you found).

1. Suppose that a rumor is spreading in the United States that the Airlines because of all the fees and other charges and potential ticket price collusion, are soon going to give away some free promotional tickets to fly anywhere in the continental U.S. Assume 20 people as of today have heard it and that it is reasonable to assume that the rumor will triple each day.

(1) Let f(t) be the function that represents the number of people that have heard the rumor t days from today. Find an equation for f(t).

(2) How many Americans will have heard the rumor 10 days from now?

(3) Predict when all Americans will have heard the rumor. Use the population as of July 5, 2015 which was 321,223,158 or go to [www.census.gov/popclock/](http://www.census.gov/popclock/) to get today’s population.

(4) Suppose instead of tripling the rumor will only double each day. Predict when all Americans will have heard the rumor.

1. The U.S. population in the 1900 Census was 76 million. In the 2000 Census, the U.S. population was 282 million. Assuming that the growth rate remained the same over that century, what year was the population of the U.S. double that from the 1900? Model the population with the equation $P=P\_{0}e^{kt}$, where $P\_{0}$ is the initial population. When you find the year the model predicts for doubling the population of the U.S., look up on the internet and see what the population actually was. Did our model predict well? Explain.
2. Suppose your new car, purchased this year for $30,000, depreciates at a rate of 12% each year.

1)When will your car be worth $10,000? Model the car value with the equation $P=P\_{0} (1-r)^{t}$, where $P\_{0}$ is the initial price value.

3)Mathematically, is it possible for your car to be worth zero dollars?

 3)Practically, is it possible for your car to be worth zero dollars?

1. Returning to part k above, The UN Population Fund states the world reached 7 billion people on Oct. 31, 2011 so it took about 12 years to go from 6 billion to 7 billion. What was the growth rate instead of the assumed 1.4% in part k?

1. Assume $20000 is invested at 4% compounded quarterly. What will it be worth in 5 years?\_\_\_\_\_\_\_\_\_\_. If you can invest it at 4% compounded continuously for the 5 years how much more interest will you make?