**Unit 5: Investigation 4 (3 Days)**

**(5 days if work from Unit 7 Algebra 1 is needed)**

**Parameters of Exponential functions**

**Common Core State Standards**

A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret as a product of *P* and a factor not depending on *P.*

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.LE.4 For exponential models, express as a logarithm the solution to where *a,c* and *d* are numbers and the base *b* is 2, 10 or *e;* evaluate the logarithm using technology.

F.BF.3Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k ( both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.LE.5 Interpret the parameters of an exponential function in terms of a context.

**Overview**:

Students who did not study unit 7 in Algebra 1 should return to investigation 3 and do all the activities 7.3.1 – 7.3.6 on graphing y = abx so that they know the roles of a and b and have used that knowledge to fit a curve to data that appears to be exponential by trial and error. They also need to do activity 7.5.6 on doubling times and half-life. This investigation starts with a problem that will be modeled with an exponential that has an exponent other than 1x, setting the stage to examine transformations on the graph of . Exponential functions of the form are rewritten as and the solutions to are expressed in in terms of logarithms.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Students will be able to apply their knowledge of the roles of k, d and c to graph y = kg(x), y = g(kx), y = d + g(x) and y = g(x + c) given . and to obtain an equation given a graph.
* Students will apply the rules from investigation 5.1 to base e and note that in particular ln ex = x and eln x = x.
* Students will understand why and apply *Any exponential function can be written in either of two forms y = abx and y = aerx where r = ln b.*
* Students will be able to solve selected applications, including continuous compound interest problems using the laws of logarithms and their ability to solve using logarithms.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 5.4.1** asks student to use their knowledge of transformations to justify a response and to use their knowledge of equivalent forms of an exponential to decide which person will have the most money in 2 years. They will need to know the finite compounding and continuous compounding formulas.
* **Exit Slip 5**.4.2 asks students to model atmospheric pressure and to find the atmospheric pressure at the top of the world’s tallest building.
* **Journal Prompt 1** Explain why the graphs of y = 4 (3)2x , y = 4(9)x , y = 4(e2.197x) appear to graph as only one curve on your grapher?
* **Activity Sheet 5.4.1 Changing Parameters** has students study the impact of changes in the graph of an exponential function based on changes in the parameters of the equation and notes that abkt = a(bk)t
* **Activity Sheet 5.4.2 Transformations Using** has students solve exponential equations and applications involving the exponential family and logarithmic families.
* **Activity Sheet 5.4.3 Exponential and Logarithmic Equations and Applications** has students solve more applied problems , finite and continuous interest, and half-life, population growth and has some exponential equations and practice changing between the two common forms of an exponential equation

**Launch Notes**:

Pose the question: Initially we have 400 bacteria. If we have a certain bacteria that is doubling every four hours, how many will there be 16 hours? How many will there be after t hours? Graph the modeling equation. How does its graph compare to the graph of the parent function? How does you graph compare with the graph of k(x) =400(20.25 )t?

**Teaching Strategies**

Have the students work in predetermined groups to answer these questions. Distribute **Activity** **sheet 5.4.1** to assist with the solution to these questions, recognizing that bkt = (bk)t and to then explore the impact of vertical and horizontal stretching and vertical and horizontal translations.

Hopefully by now students should be able to anticipate the graphical changes by looking at an equation since they have done this for linear, quadratic, general polynomial, rational, and logarithmic families. They should where appropriate be able to defend the shape of their graph based on abkt = a(bk)t

**Group Activity : Activity sheet 5.4.1** will have students solve several problems, perhaps by table, noting patterns and make generalizations about the impact of a, b, c and d on the graph of f(x) = d + abcx .

**Activity Sheet 5.4.2** will have students graph y = ex, and note that for any output b we can find an x value, say r such that er = b where of course r = ln b . It might help for students to graph y = ex and y = b for several values of b where b > 0 and note the two curves always intersect and for the output b there is always an input value. We use the inverse function to express that input value explicitly as ln b so that y = abx =a (er)x = aerx = a exln b . *The big idea is that any exponential function can be written in either of two forms y = abx and y = aerx where r = ln b.*

Some continuous compounding problems are included as well as other applications.

**Differentiated Instruction (For Learners Needing More Help** ) Before beginning Activity 5.4.2 students may need to practice the raising to a power rule with concrete examples such as ((2)3)4 = 2 12

**Differentiated Instruction (For Learners Needing More Help)**  Use technology and have students graph a number of pairs y = ex and y = b, use the intersect command to obtain the coordinates (approximately) of the point of intersection, (r, b). Have students interchange the coordinates and stress these would be coordinates on a logarithm graph, (b, r) where r = ln b. So if we have y = abx we can rewrite as y = ae ln b x.

**Differentiated Instruction (Enrichment)** For students who did function composition in unit 1 have them verify ln ex = x and eln x = x if they did not do Activity 5.1.5B. If they did 5.1.5B they should be able to explain why these statements therefor follow.

**Journal Prompt 1**

Explain why the graphs of y = 4 (3)2x , y = 4(9)x , y = 4(e2.197x) appear to graph as only one curve on your grapher? A response might be: (3)2x = (32)x = 9x by a Law of Exponents and the ln 9 is approximately 2.197 and e ln 9 = 9 is a basic law of logarithms or STEM intending should say they are inverses.

A problem that has several exponential functions with different bases needs us to change to one base if we desire an exact answer and not an approximation. **Exit slip 5.4.1** can be assigned any time after Activity 5.4.2 has been discussed. Can we solve 400 (1.4 )x = 300(1.5)x?

**Activity sheet 5.4.3** will have application problems and equations to solve using for the most part base10 or base *e*. Students can now solve half-life problems and doubling times problems explicitly instead of using trial and error as they did in Algebra 1. There are more problems finding finite and continuous interest and applications that student must consider first—is a continuous model more appropriate. . **Exit slip 5.4.2** can be used after Activity 5.4.3 has been discussed

**Closure Notes**

This a a good time to review the impact of reflections across the axes, vertical shifts and stretches, and horizontal shifts and stretches. Students will be applying this knowledge to the trigonometric family in the next unit.

**Vocabulary**

Continuous compound

Doubling Time

Finite compounding

Half-Life

Raising to a Power Rule for Exponents

**Resources and Materials**

**Activities 5.4.1 – 5.4.3 should be completed. However in Activity 5.4.3 not every application needs to be assigned.**

Activity Sheet 5.4.1 Changing Parameters

Activity Sheet 5.4.2 Transformations Using

Activity sheet 5.4.3 Exponential and Logarithmic Equations and Applications

Graphers or equivalent technology