**Activity 2.1.5 How Do Quadratic Functions Behave?**

Quadratic functions give us an opportunity to investigate a function that does not have the constant form that linear functions have. A popular vocabulary term from linear functions is slope. Slope describes a constant rate of change and as such, is a very accurate descriptor to get a visual comprehension of what the linear function looks like before you sketch it.

In this investigation, we will introduce and develop a new set of terms that will help us do the same thing for quadratic functions.

***The objective:*** Using the symbolic form of the quadratic function (Standard Form or Vertex Form), we want to be able to describe the key point of the quadratic function and its behavior.

***Key Terms:***

**x-intercept y-intercept increasing function decreasing function**

**axis of symmetry concave up concave down end behavior**

1. Using technology graph the following function:

 Window settings: xmin = -2, xmax = 8, xscl = 1

 ymin = 0, ymax = 40, yscl = 5

We know from prior investigations that this function is a transformed version of the parent function. Since it is written in vertex form, we can identify a couple of features right away. When appropriate, circle the correct choice or fill in the blank.

The value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_\_.

This tells us that this function opens *up/down* because the value of a is *positive/negative*.

The width of this function compared to the parent function is *wider/narrower* because the absolute value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_ which is *greater/less* than 1.

Since we know that the function opens *up/down*, we can also refer to the parabola as ***concave up/down***.

*Now we turn to some other points that you already should be familiar with. Keep in mind that x-intercepts are the point(s) (x, y) when y = 0 and y-intercept is the point (x, y) when x = 0.*

How many x-intercepts does this function have? \_\_\_\_ Using the picture you have, please describe how you know that? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

How many y-intercepts does this function (parabola) have? \_\_\_\_ Again, using the picture you have, please describe how you know that? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Identify the vertex in coordinate form \_\_\_\_\_\_\_\_\_\_\_\_

Identify the axis of symmetry: the line with equation x = \_\_\_\_\_

As you follow the parabola from left to right in your window notice how the y value changes. From the far left, the values are *decreasing*. Then, you hit a bottom at the vertex. After that, the y values begin to *increase*. Let’s work on a rule for this description. As usual, this concept will come up in later units when we describe even and odd polynomials, so you’ll want to understand this conversation, not memorize it! We will use x-values as our independent values. As we move along the x-axis from negative infinity  towards the vertex, our function is *decreasing*. At the vertex, our function is neither increasing nor decreasing. As we continue to move right along the x-axis, after the vertex, towards positive infinity , our function is *increasing*. This behavior is indicative of all parabolas and demonstrates how the quadratic function family is not a family of constant growth like the family of linear functions. There are many real world phenomena whose mathematical models behave in this parabolic manner and the ability to grasp the patterns generically will aid in our problem solving ability in later investigations.

**Let’s practice more and see how we are doing!** Fill in the blanks using the word choices that were previously provided for you in question 1.

2. Using technology graph the following function:

 Window settings: xmin = -12, xmax = 3, xscl = 1

 ymin = -3, ymax = 27, yscl = 2

The value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_\_.

This tells us that this function opens \_\_\_\_\_\_\_\_\_\_\_\_ because the value of *a* is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The width of this function compared to the parent function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ because the absolute value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_ which is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than 1.

Since we know that the function opens \_\_\_\_\_\_\_\_\_\_\_\_, we can also now refer to the parabola as ***concave \_\_\_\_\_\_\_\_\_\_\_\_***.

How many x-intercepts does this function have? \_\_\_\_ Using the picture you have, please describe how you know that? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Using the graph or the table of values identify an estimate for the x-intercepts as coordinate pairs. \_\_\_\_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_\_\_\_

Identify the y-intercept as a coordinate pair \_\_\_\_\_\_\_\_\_\_\_

Identify the vertex as a coordinate pair \_\_\_\_\_\_\_\_\_\_\_

Identify the axis of symmetry: the line with equation x = \_\_\_\_\_

As you follow the parabola from left to right in your window describe the changes in the y-values as we did in #1:

As we move along the x-axis from negative infinity  towards the vertex, our function is *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*.

At the vertex, our function is neither increasing nor decreasing.

As we continue to move right along the x-axis, after the vertex, towards positive infinity , our function is *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*.

This description is symbolically written as the following:

As  ; as **

The tough part here is that the symbolic way of writing the information deviates from our “left to right” approach. It will take a bit of adjusting until you feel comfortable with it. That’s ok, just continue to practice and hang in there!

3. Using technology graph the following function:

 Window settings: xmin = -10, xmax = 10, xscl = 1

 “zoom standard” ymin = -10, ymax = 10, yscl = 1

The value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_\_.

This tells us that this function opens \_\_\_\_\_\_\_\_\_\_\_\_ because the value of *a* is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The width of this function compared to the parent function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ because the absolute value of a = \_\_\_\_\_\_\_\_\_\_\_\_ which is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than 1.

Since we know that the function opens \_\_\_\_\_\_\_\_\_\_\_\_, we can also now refer to the parabola as ***concave \_\_\_\_\_\_\_\_\_\_\_\_***.

How many x-intercepts does this function have? \_\_\_\_ Using the picture you have, please describe how you know that? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Using the graph or the table of values identify an estimate for the x-intercepts as coordinate pairs. \_\_\_\_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_\_\_\_

Identify the y-intercept as a coordinate pair \_\_\_\_\_\_\_\_\_\_\_

Identify the vertex as a coordinate pair \_\_\_\_\_\_\_\_\_\_\_

Identify the axis of symmetry: the line with equation x = \_\_\_\_\_

As you follow the parabola from left to right in your window describe the changes in the y-values as we did in #1:

As we move along the x-axis from negative infinity  towards the vertex, our function is *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*.

At the vertex, our function is neither increasing nor decreasing.

As we continue to move right along the x-axis, after the vertex, towards positive infinity , our function is *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*.

This description is symbolically written as the following:

As  ; as **

Ok, now let’s try a couple in standard form!

4. Using technology graph the following function:

 Window settings: xmin = -5, xmax = 5, xscl = 1

 ymin = -10, ymax = 10, yscl = 1

The value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_\_.

This tells us that this function opens \_\_\_\_\_\_\_\_\_\_\_\_ because the value of *a* is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The width of this function compared to the parent function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ because the absolute value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_ which is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than 1.

Since we know that the function opens \_\_\_\_\_\_\_\_\_\_\_\_, we can also now refer to the parabola as ***concave \_\_\_\_\_\_\_\_\_\_\_\_***.

How many x-intercepts does this function have? \_\_\_\_ Using the picture you have, please describe how you know that? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Using the graph or the table of values identify an estimate for the x-intercepts as coordinate pairs. \_\_\_\_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_\_\_\_

Identify the y-intercept as a coordinate pair \_\_\_\_\_\_\_\_\_\_\_ (***hint:*** Notice the value of c!)

Identify the vertex as a coordinate pair \_\_\_\_\_\_\_\_\_\_\_ (***hint:*** vertex in standard form is 

Identify the axis of symmetry: the line with equation x = \_\_\_\_\_

As you follow the parabola from left to right in your window describe the changes in the y-values as we did in #1:

As we move along the x-axis from negative infinity  towards the vertex, our function is *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*.

At the vertex, our function is neither increasing nor decreasing.

As we continue to move right along the x-axis, after the vertex, towards positive infinity , our function is *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*.

This description is symbolically written as the following:

As  ; as **

5. Using technology graph the following function:

 Window settings: xmin = -15, xmax = 5, xscl = 1

 ymin = -5, ymax = 25, yscl = 2

The value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_\_.

This tells us that this function opens \_\_\_\_\_\_\_\_\_\_\_\_ because the value of *a* is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The width of this function compared to the parent function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ because the absolute value of *a* = \_\_\_\_\_\_\_\_\_\_\_\_ which is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than 1.

Since we know that the function opens \_\_\_\_\_\_\_\_\_\_\_\_, we can also now refer to the parabola as ***concave \_\_\_\_\_\_\_\_\_\_\_\_***.

How many x-intercepts does this function have? \_\_\_\_ Using the picture you have, please describe how you know that? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Using the graph or the table of values identify an estimate for the x-intercepts as coordinate pairs. \_\_\_\_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_\_\_\_

Identify the y-intercept as a coordinate pair \_\_\_\_\_\_\_\_\_\_\_ (***hint:*** Notice the value of *c*!)

Identify the vertex as a coordinate pair \_\_\_\_\_\_\_\_\_\_\_ (***hint:*** vertex in standard form is 

Identify the axis of symmetry: the line with equation x = \_\_\_\_\_

As you follow the parabola from left to right in your window describe the changes in the y-values as we did in #1:

As we move along the x-axis from negative infinity  towards the vertex, our function is *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*.

At the vertex, our function is neither increasing nor decreasing.

As we continue to move right along the x-axis, after the vertex, towards positive infinity , our function is *\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*.

This description is symbolically written as the following:

As  ; as **