**Activity 2.3.3 Complex Numbers**

Now that you know that *i* is the imaginary number (often referred to as the imaginary unit ) equivalent to , and that *i* is not a real number, you are ready to put real numbers together with imaginary numbers to obtain *Complex Numbers.* An example of a complex number is 2 + 3*i*. The real part is the real number 2, the imaginary part is the 3 that multiplies the imaginary number *i.* Complex numbers are of the form a + bi : the real part is \_a\_ the imaginary part is \_b\_

The arithmetic of the complex numbers is much like the reals but there is the twist : *i2* is always equal to -1, and you must always write the square root of a negative number as an imaginary number before performing any operations. For example, write as *i* before operating with . Some other theorems like we saw in the previous activity do not hold.

Start with what you know about real numbers to reason how to operate with Complex Numbers:

1. Addition and Subtraction of Complex numbers:

Show you know how to do this addition:

a. (3 + 2x) + (5 - 3x) = \_\_\_\_\_\_

Now try this. Be sure to write your answer in a + bi form even if the real or imaginary part is 0.

b. (3 + 2i) + (5 - 3i) c. (3 + 2i) + (3 - 2i)

d. (8 – 2i) – (-2 + 13i) -10 e. 3i – (3 + 2i)

2. Multiplication of Complex Numbers

a. Show you know how to do this multiplication: (3 + 2x)(5 - 3x)

Now try multiplying these complex numbers. It is similar to the multiplication in part a, but the extra piece you have to remember is to rename *i2* as -1, then simplify. As before, write your answer in a + bi form.

b. (3 + 2i)(5 - 3i) c. (3 + 2i)(3 - 2i)

d. 10(8 – 2i) (-2 + 13i) e. (3i)(3 + 2i)

3. Conjugate of a Complex Number

The conjugate of a + bi is a - bi and is denoted with a bar over the top:

“z” is often used to denote a + bi, so z-bar (written z ) is the conjugate of z.

The conjugate of a complex number is the complex number that has the same real part and the opposite of the imaginary part.

3 + 5i: The real part is\_\_\_ the imaginary part is \_\_\_ The conjugate of 3 + 5i is \_\_\_\_\_\_

a. 4 - 6i: The real part is\_\_\_\_ the imaginary part is \_\_\_\_ The conjugate of 4 - 6i is­­­­­­­­­­­­­­­­­\_\_\_\_\_\_\_

b. -3 - 7i:The real part is\_\_\_ the imaginary part is \_\_\_ The conjugate of -3 - 7i is \_\_\_

c. -8i : The real part is\_\_\_ the imaginary part is \_\_\_ The conjugate of -8i is \_\_\_\_\_

d. 9 : The real part is\_\_\_ the imaginary part is \_\_\_ The conjugate of 9 is \_\_\_\_\_

e. When you did part d., did you notice something? If a number equals its complex conjugate, then it is not only a complex number, but also a \_\_\_\_\_ number.

f. Add a complex number and its conjugate:

(3 + 5i) + (3 - 5i) (-5 - 7i) + (-5 + 7i)

What kind of number do you obtain when you add a complex number to its conjugate?

\_\_\_\_\_\_\_\_

g. Multiply a complex number and its conjugate:

(3 + 5i)(3 - 5i) (-5 - 7i)(-5 + 7i)

What kind of number do you obtain when you multiply a complex number by its conjugate?

\_\_\_\_\_\_\_\_

h. True or false: Complex numbers are closed under addition, subtraction and multiplication.

4. Dividing Complex Numbers

 a. Warm up: Rewrite the following expressions by eliminating radicals in the denominator.

3 ÷ = = and this one: 3 ÷ =

Like any fraction you have met before, the way to find an equivalent fraction is the use the multiplicative identity 1. We can have a real number in the denominator but not a complex number that is not also real when dividing complex numbers. If our fraction was we could write it as and we would be done.

Think about how we solved the issue of rationalizing the denominators above.

 b. Divide: 3 ÷ 5i = =

 c. Divide: 1 ÷ i =

1. Divide:
2. Divide: 3 ÷ 5i = =

f. Divide: 3 ÷) = =

(Hint: This has two terms in the denominator so multiply the denominator by the conjugate.)

g. Now consider the quotient . Our answer must be in the form a + b*i*.

Think about how we solved the issue of rationalizing the denominators above when the denominator had 2 terms in the warm up and number f.

The key is to multiply by . Conjugates to the rescue!!!!!

Try it.

Now do these: write final answer in a + bi form

h. Divide: 3 ÷ (2 - 5i) =

1. Divide:
2. Divide:
3. Divide

l. Are the Complex Numbers closed under division? Explain.

1. Note: Rationalizing denominators is a useful technique, but there is nothing wrong with having a radical in the denominator, except in high school math competitions where you are told to write radicals in “simplified form”. In fact, in calculus, it will be useful to rationalize the numerator.

If you cannot use a calculator, which expression is easier to simplify by hand

 where √3?

Rationalize the numerator

Challenge/History: Why do people consider rationalized denominators to be simplified form? Hint: consider the different scales on a slide rule. Can you divide a square root by another square root on the slide rule?

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