**Activity 5.2.2 The Radius-Tangent Theorem**

In Activity 5.4.1 we concluded that it appeared that if a line is tangent to a circle then it is perpendicular to the radius at the point of tangency. Our task is now to prove this.

1. We begin with another theorem: **Shortest Distance Theorem:** The shortest segment joining a point to a line is the perpendicular segment.

By our construction from Unit 2 Investigation 7 we know that we can construct a line perpendicular to $\overleftrightarrow{AB}$ through the point *C* and call the point of intersection *D*.

a. Construct segment $\overbar{CB}$. Sketch this segment on the diagram above.

b. Explain why $∠$ *CDB* is the largest angle in ∆ *DCB*.

c. Fill in the blank with > , < or =: m$∠$ *CDB* \_\_\_\_\_\_\_ m $∠$ *CBD*.

d. Now what can we conclude about sides?

 Fill in the blank with > , < or =: *CB* \_\_\_\_\_\_\_ *CD*.

e. What theorem supports your conclusion in question 1d?

Since *B* was any point on the line other than *D*, we can conclude that **the shortest segment joining a point to a line is the perpendicular segment.**

1. We are now ready to address our original question. What is the relationship between a radius of a circle and a tangent to the end of the radius on the circle? We will consider this in two parts. First we will establish a tangent at a point, then we will establish that any tangent is perpendicular to the radius.



We begin with a radius ($\overbar{AB}$) of a circle and construct a line perpendicular to $\overbar{AB}$ at *B*. We choose a point *C* to be another point on the perpendicular line.

a. Draw segment $\overbar{AC}$. We note that *AC*>*AB.*
Why?

This must mean that *C* lies outside the circle. Because *C* could be any point on the perpendicular line other than *B*, the only intersection of$\overleftrightarrow{BC}$ with the circle is point *B*.

b. Because $\overleftrightarrow{BC }$ intersects the circle in exactly \_\_\_\_\_point, it is a \_\_\_\_\_\_\_\_\_\_\_ to the circle. This means that a line perpendicular to a radius at a point of a circle is a tangent.



1. Now we begin with a tangent line $\overleftrightarrow{BC}$ a circle *A* at a point *B* and show that it is perpendicular to the radius $\overbar{AB}$*.*

Suppose $\overbar{AB}$ is not perpendicular to $\overleftrightarrow{BC}$. Then draw $\overbar{AD}$ so that $\overbar{AD}$ $⊥\overleftrightarrow{BC}$ at point *D.* Reflect *B* over $\overbar{AD}$ and name its image *B*’.

1. *B*’ lies on $\overleftrightarrow{BC}$. Why?
2. *AB* = *AB*’. Why?
3. *B*’ lies on circle *A*. Why?

If our assumption that $\overbar{AB}$ is not perpendicular to $\overleftrightarrow{BC}$ is true, then line $\overleftrightarrow{BC}$ would intersect circle *A* in two points, *B* and *B*’, and could not be a tangent to circle *A.* But we were given that $\overleftrightarrow{BC}$ is tangent to the circle. Therefore our assumption that $\overbar{AB }$is not perpendicular to $\overleftrightarrow{BC}$ is false and we conclude that $\overbar{AB}$ must be perpendicular to $\overleftrightarrow{BC}$.