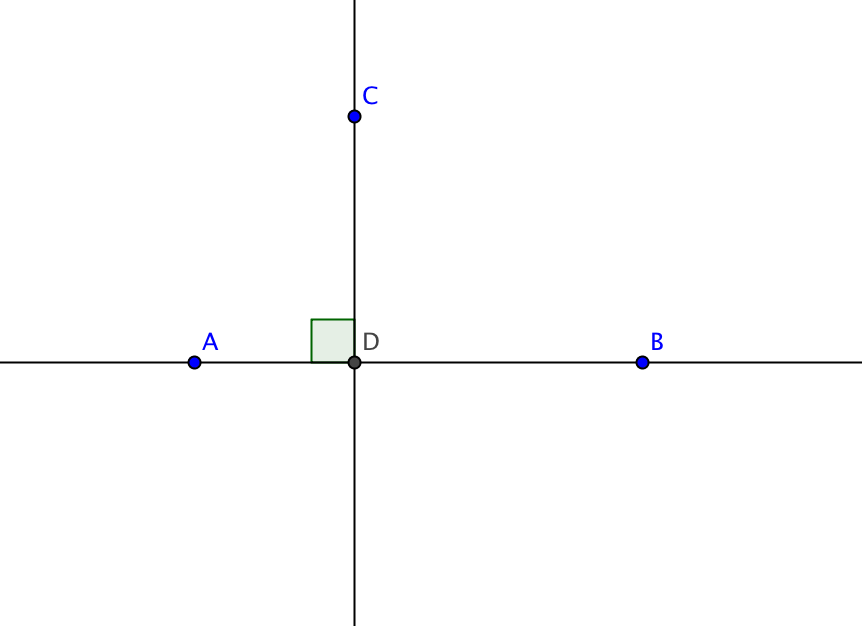
**Activity 5.2.2 The Radius-Tangent Theorem**

In Activity 5.4.1 we concluded that it appeared that if a line is tangent to a circle then it is perpendicular to the radius at the point of tangency. Our task is now to prove this.

1. We begin with another theorem: **Shortest Distance Theorem:** The shortest segment joining a point to a line is the perpendicular segment.  
     
   By our construction from Unit 2 Investigation 7 we know that we can construct a line perpendicular to through the point *C* and call the point of intersection *D*.

a. Construct segment . Sketch this segment on the diagram above.

b. Explain why *CDB* is the largest angle in ∆ *DCB*.

c. Fill in the blank with > , < or =: m *CDB* \_\_\_\_\_\_\_ m *CBD*.

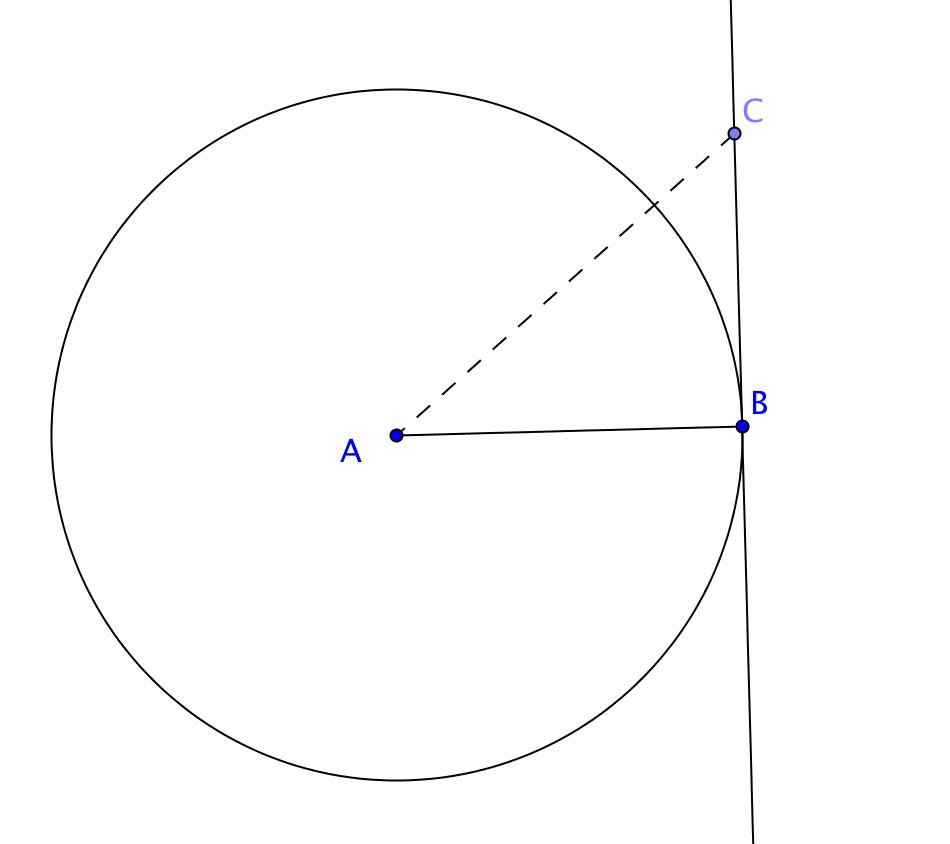
d. Now what can we conclude about sides?

Fill in the blank with > , < or =: *CB* \_\_\_\_\_\_\_ *CD*.

e. What theorem supports your conclusion in question 1d?

Since *B* was any point on the line other than *D*, we can conclude that **the shortest segment joining a point to a line is the perpendicular segment.**

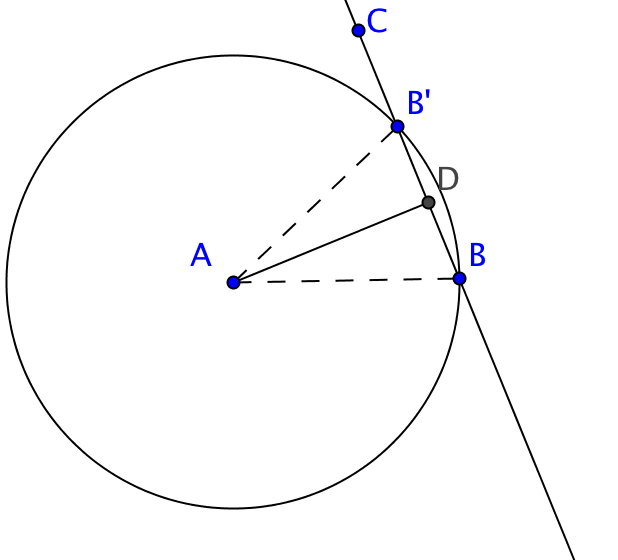
1. We are now ready to address our original question. What is the relationship between a radius of a circle and a tangent to the end of the radius on the circle? We will consider this in two parts. First we will establish a tangent at a point, then we will establish that any tangent is perpendicular to the radius.



We begin with a radius () of a circle and construct a line perpendicular to at *B*. We choose a point *C* to be another point on the perpendicular line.

a. Draw segment . We note that *AC*>*AB.*   
Why?   
  
This must mean that *C* lies outside the circle. Because *C* could be any point on the perpendicular line other than *B*, the only intersection of with the circle is point *B*.

b. Because intersects the circle in exactly \_\_\_\_\_point, it is a \_\_\_\_\_\_\_\_\_\_\_ to the circle. This means that a line perpendicular to a radius at a point of a circle is a tangent.



1. Now we begin with a tangent line a circle *A* at a point *B* and show that it is perpendicular to the radius *.*

Suppose is not perpendicular to . Then draw so that at point *D.* Reflect *B* over and name its image *B*’.

1. *B*’ lies on . Why?
2. *AB* = *AB*’. Why?
3. *B*’ lies on circle *A*. Why?

If our assumption that is not perpendicular to is true, then line would intersect circle *A* in two points, *B* and *B*’, and could not be a tangent to circle *A.* But we were given that is tangent to the circle. Therefore our assumption that is not perpendicular to is false and we conclude that must be perpendicular to .