**Activity 5.8.6 Eccentricity of the Conic Sections**

Recall the locus definition of parabola: A parabola is the locus of points that are equidistant from a fixed point (called the focus) and a fixed line (called the directrix.)

Let’s modify our definition a bit and consider a curve that is the locus of points where the ratio between its distance to the focus and its distance to the directrix is a fixed positive number *e*.

(Recall from Activity 5.8.4 that for an ellipse, *e* = $\frac{c}{a}$ is defined as its eccentricity. We’ll show that the “*e*” we are using here is the same thing.)

What will this locus look like? And how will changing *e* affect the shape of the curve?

Open the file ctcoregeomACT586 and experiment. Note the instruction in the file and in the box below.

The directrix is located at *y* = –1

The focus is locate at (0,*e*) where *e* is the eccentricity.

The slider for *d* will give you points that are *d* units from the directrix

and *ed* units from the focus.

Move the slider to generate the curve.

To erase the trace go to “Refresh Views” in the view menu.

Change the eccentricity by typing *e* = whatever in the input window.

Experiment with different values of *e* and observe what happens

After performing your experiments, answer these questions:

1. If the point on the locus is *ed* units from the focus and *d* units from the directrix, what is the ratio of the two distances? $\frac{Distance from focus}{Distance from directrix}$ = \_\_\_\_\_\_\_\_.
2. If *e* > 1 what type of curve do we have?
3. If *e* = 1, what type of curve do we have?
4. If 0 < *e <* 1 what type of curve do we have?
5. What happens when *e* = 0?
6. Derive an equation for the locus from the definition given above:
Let the coordinates of the point on the locus be (*x*, *y*). Then
Distance from point to focus = *e*$ ∙$ distance from point to directrix

 $\sqrt{(x-0)^{2}+(y–e)^{2}}$ = *e*(*y*+1)

 Square both sides and simplify the equation.

1. Let *e* = 1. Show that the equation you derived in question 6 gives you the standard equation for a parabola.
2. If *e* ≠ 1, our experiments suggest that we have an ellipse or a hyperbola. How are these curves different from ellipses and hyperbolas in standard position?
3. If the major axis of an ellipse runs along the *y­-*axis, then it can be shown that its equation is of the form $\frac{(y-k)^{2}}{a^{2}}+\frac{x^{2}}{b^{2}}=1,$ where (0, *k*) is the center of the ellipse, 2*a* is the length of the major axis, and 2*b* is the length of the minor axis.

a. In the ellipse pictured at the right the center is (0,1) and the foci are at ($0,\frac{1}{2})$ and ($0,\frac{3}{2})$.
The length of the major axis is 2*a*. Therefore *a* = \_\_\_\_
The distance from the center to a focus = *c* = \_\_\_\_\_\_
We can find *b* using the equation *a*2 = *b*2 + *c*2. *b* = \_\_\_\_\_\_

b. Now substitute into $\frac{(y-k)^{2}}{a^{2}}+\frac{x^{2}}{b^{2}}=1$ to find an equation for this particular ellipse.

c. This ellipse can also be generated using focus-directrix definition introduced earlier in this investigation. In this case the directrix is the line *y* = – 1 and the focus *F*1 is (0, \_\_).
What is the eccentricity of this ellipse?

d. Substitute for *e* the equation you found in question 6. Then show that this agrees with the equation found in question 9b.

1. If the transverse axis of a hyperbola coincides with the *y­-*axis, then it can be shown that its equation is of the form $\frac{(y-k)^{2}}{a^{2}}-\frac{x^{2}}{b^{2}}=1,$ where (0, *k*) is the center of the figure, *c* is the distance from the center to each focus, 2*a* is the distance between the *y*-intercepts, and
 *b*2 = *c*2 – *a*2.
2. In the hyperbola pictured at the right the center is (0,–2) and the foci are at ($0, 2)$ and (0, ­–6$)$. The distance between the *y-*intercepts, 2*a* = \_\_\_\_, so *a* = \_\_\_\_
The distance from the center to a focus = *c* =\_\_\_\_\_
We can find *b* using the equation *b*2 = *c*2 – *a*2.
*b* = \_\_\_\_\_\_.
3. Now substitute into $\frac{(y-k)^{2}}{a^{2}}-\frac{x^{2}}{b^{2}}=1$ to find an equation for this particular hyperbola. Solve it for $x^{2}.$
4. This hyperbola can also be generated using focus-directrix definition introduced earlier in this investigation. In this case the directrix is the line *y* = – 1 and the focus *F*1 is (0, \_\_). What is the value of *e* is we were to general the hyperbola as we did in question in the experiments? (This value is the eccentricity of the hyperbola.)
5. Substitute for *e* the equation you found in question 6. Then show that this agrees with the equation found in question 9b.
6. The focus-directrix definition only gives us the upper branch of the hyperbola. The lower branch is generated by using *F*2 (0, –6) as the focus. Find the equation of the directrix associated with this focus. Explain your reasoning.