**Activity 7.5.3 – Density Curves and Normal Curves**

There are many situations in which a random variable can take on *any* value in a certain interval. When this occurs, we say that the random variable is *continuous*. This activity examines how continuous random variables can be modeled by curves.

**Frequency Distribution**

**Heights of 16-Year Old Girls**

|  |  |  |
| --- | --- | --- |
| **Class** | **Frequency** | **Relative****Frequency** |
| $$58\leq x<60$$ | 32 | 0.08 |
| $$60\leq x<62$$ | 44 | 0.11 |
| $$62\leq x<64$$ | 72 | 0.18 |
| $$64\leq x<66$$ | 92 | 0.23 |
| $$66\leq x<68$$ | 80 | 0.20 |
| $$68\leq x<70$$ | 60 | 0.15 |
| $$70\leq x<72$$ | 20 | 0.05 |

The frequency distribution to the right summarizes the heights (in inches) of four hundred 16-year old teenage girls. The random variable *x* represents the height of a teenage girl in this distribution.

The relative frequency histogram below displays this distribution.

**Relative Frequency Histogram**



The relative frequency histogram shows the proportion of heights in each class. We can convert this histogram to a *density histogram* by changing the height of each bar to its density value. A bar’s density value is its relative frequency divided by its width. Changing the height of the bar from relative frequency to density makes the *area* of each bar equal to the relative frequency. The histogram below is a density histogram. The value above each bar is the bar’s density.

**Density Histogram**



Density histograms enable us to determine the percentage of random variable values within certain *intervals.* The percentage of values within an interval equals the probability that a randomly selected value is in the interval.

Use the density histogram below to answer the following questions.



1. Find the area of the bar between 58 and 60. What does this value represent?
2. What is the total area of all the bars? What does this represent?
3. What percent of heights are less than 62 inches?
4. What percent of heights are between 62 and 68 inches?
5. What percent of heights are greater than 68 inches?
6. What is more likely: randomly choosing a student whose height is greater than 68 inches or randomly choosing a student whose height is less than 62 inches? Explain.
7. Can we use the density histogram to determine the percent of heights between 61 and 63 inches? Explain.

**Density Curves**

The density histogram details areas for a finite number of regions – the regions that relate to the bars in the histogram. When we want to examine the behavior of a continuous random variable – a random variable that can assume *any* value in an interval – we need a more detailed and flexible model. A *density curve* is such a model.

1. Sketch a density curve on the density histogram above. Draw a curve that begins and ends near the *x*-axis and intersects the top of the bars.



The density curve below models the density histogram. This curve allows us to estimate the percent of heights within *any* interval. The image below shows the percent of heights between 61 and 63 inches (labeled A), and the percent of heights greater than 69 inches (labeled B).

 

The area below the density curve within an interval *approximates* the actual percentage of data values within the interval. Sometimes the area below the curve overestimates; sometimes it underestimates.

**Area** Below Density Curve within an Interval

= **Percentage** of Values Within the Interval

= **Probability** that a Randomly Selected Value is Within the Interval

**Modeling a Distribution of Exam Scores**

The density histogram below displays the exam scores for middle school students on a standardized exam.



1. What percent of exam scores are between 40 and 70?
2. What percent of exam scores are between 70 and 90?

1. Sketch a density curve on the density histogram above. Draw a curve that begins and ends near the *x*-axis and intersects the top of the bars.
2. Show the area below the density curve that corresponds to the percent of scores that are less than 36. What geometric shape can we use to find this area? Estimate the area.
3. Show the area below the density curve that corresponds to probability that a randomly selected exam score is between 75 and 93. What geometric shape can we use to find this area? Estimate the area.

**Continuous Probability Distributions**

Density curves are *continuous probability distributions* – models of continuous random variables that can be used to describe the behavior of random variables. These models are effective tools for describing distributions and finding probabilities associated with random events.

**Normal Curves**

Normal curves are density curves (continuous probability distributions) that model symmetric, bell-shaped distributions.

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 $μ-3σ μ-2σ μ-σ μ μ+σ μ+2σ μ+3σ$

Normal curves have the following properties:

* $μ$ is the mean of the distribution and $σ$ is the standard deviation of the distribution.
* The total area below the curve is 1.
* The curve extends endlessly in both directions, approaching, but never touching, the *x*-axis.
* 68% of the area under the curve is between $μ-σ$ and $μ+σ; $95% of the area under the curve is between $μ-2σ$ and $μ+2σ; $and 99.7% of the area under the curve is between $μ-3σ$ and $μ+3σ$.

The image below displays two normal curves.

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1. Estimate the mean of each curve.
2. Which curve has a greater standard deviation?

**Modeling Data Distributions**

The density histogram below displays five hundred middle-school students typical number of hours of sleep per night.

 

1. Can this distribution be modeled well by a normal curve? Explain.
2. Estimate the mean and standard deviation of the distribution.
3. Sketch a normal curve on the density histogram above. Draw a curve that begins and ends near the *x*-axis and intersects the top of the bars.
4. Show the area below the normal curve that corresponds to the percent of random variable values that are less than 7.8. Use geometry to estimate this percentage.
5. Show the area below the normal curve that corresponds to the probability that a randomly selected value is between 8.8 and 10.5. Use geometry to estimate this probability.
6. What values of the random variable could be considered unusual?