**Activity 7.5.5 – Inference with Normal Curves**

Normal curves are powerful models for describing the behavior of continuous random variables. In this activity we will use normal curves to model sampling distributions of sample means and find probabilities to perform statistical inference on unknown population means.

**Central Limit Theorem for Sample Means**

If random samples of size *n* are repeatedly taken from a population that is normally distributed with mean $μ$ and standard deviation $σ$, the sampling distribution of sample means will also be normally distributed.

The mean of the sampling distribution of sample means is the population mean, and the standard deviation of the sampling distribution of sample means is the population standard deviation divided by the square root of the sample size.

$$Mean:μ\_{\overbar{x}}=μ Standard Deviation:σ\_{\overbar{x}}= \frac{σ}{\sqrt{n}}$$

Recall that a sampling distribution of sample means is the distribution of *all* possible sample means from random samples of the same size. The standard deviation of sample means is also called the *standard error*.

**SAT Mathematics Scores**

In a recent year SAT mathematics scores of high school students in a state were approximately normally distributed with a mean $μ=510$ and a standard deviation $σ=120$.

Suppose we take random samples of size *n* = 30 from this population.

1. Find the mean and standard deviation of sample means from random samples of size *n* = 30.
2. Since the population of SAT scores is normally distributed, sample means from random samples of size *n* = 30 are *also* normally distributed. Sketch the sampling distribution of sample means below. Indicate the locations of the mean and values that are one, two, and three standard errors above and below the mean.

Sampling Distribution of Sample Means, *n* = 30



1. How does the distribution of sample means from random samples of size *n* = 30 compare to the distribution of actual SAT mathematics scores?

Using the mean and standard error of sample means from random samples of size *n* = 30, we can find the probability associated with sample means, $\overbar{x}$.

In Questions 4 and 5, shade in the area corresponding to probability on the normal curve you made in number 2, and find the probability.

1. What is the probability of obtaining a sample mean, $\overbar{x}$, less than 470?
2. What is the probability of obtaining a sample mean, $\overbar{x}$, greater than 560?
3. According to the Empirical Rule, what sample means are considered unusual?

Probability distributions for sample means enable us to use sample means to test claims about population means. The procedure below outlines the process of performing a hypothesis test on a claim about a population mean using a normal model for a sampling distribution.

**Hypothesis Test For a Population Mean (Using a Normal Probability Distribution)**

1. Make an assumption about a population mean (hypothesis)
2. Construct a sampling distribution of sample means (normal distribution) using the assumed value of the population mean and an assumed value of the population standard deviation
3. Find the probability of observing a sample mean as extreme as the one found
4. State a conclusion about the population mean

**Breakfast Cereals**

A nutritionist claims that breakfast cereals sold in Connecticut supermarkets contain an average of 160 mg of sodium per serving (3/4 cup). You obtain a random sample of 24 breakfast cereals from Connecticut supermarkets and find that it has a mean of 170 mg of sodium per serving.

Does this sample provide evidence that the mean sodium content of breakfast cereals sold in Connecticut supermarkets is greater than 160 mg?

Let’s assume the per-serving sodium amounts in breakfast cereals is normally distributed with standard deviation $σ=20$ mg.

1. State the assumption (hypothesis) about the population mean.
2. Assuming the hypothesis about the population mean is true and the population standard deviation is $σ=20$, determine the mean and standard deviation of the sampling distribution of sample means for random samples of size *n* = 24.
3. Sketch the sampling distribution of sample means below. Indicate the locations of the mean and values that are one, two and three standard errors from the mean.

Sampling Distribution of Sample Means, *n* = 24



1. The *observed* sample mean is $\overbar{x}=1$70. Indicate the position of this sample mean on the sampling distribution above.
2. Find and interpret the *z*-score of the observed sample mean.
3. Use the sampling distribution to find the probability of obtaining a random sample of size

*n* = 24 with a sample mean $\overbar{x} $of 170 or more. This probability is the *P*-value for this hypothesis test.

1. Is the observed sample mean $\overbar{x}=1$70 statistically significant? Explain.
2. State a conclusion about the assumed value of the population mean.
3. What can we conclude about the mean per-serving sodium content of breakfast cereals sold in Connecticut supermarkets?

**Efficiency of Hybrid Vehicles**

A car manufacturer claims that its hybrid cars have a mean highway gas mileage of 50 miles per gallon (mpg). A researcher obtains a random sample of *n* = 25 hybrid cars from the manufacturer and finds a sample mean of 48 miles per gallon.

Does this sample provide evidence that the mean highway gas mileage for all of the manufacturer’s hybrid vehicles is less than 50 miles per gallon?

Let’s assume the highway gas mileages of these cars are normally distributed with a standard deviation $σ=8$ miles per gallon.

1. State the assumption (hypothesis) about the population mean.
2. Assuming the hypothesis about the population mean is true and the population standard deviation is $σ=8$, determine the mean and standard deviation of the sampling distribution of sample means for random samples of size *n* = 25.
3. Sketch the sampling distribution of sample means below. Indicate the locations of the mean and values that are one, two, and three standard errors from the mean.

Sampling Distribution of Sample Means, *n* = 25



1. The *observed* sample mean is $\overbar{x}=48$. Indicate the position of this sample mean on the sampling distribution above.
2. Find and interpret the *z*-score of this sample mean.
3. Use the sampling distribution to find the probability of obtaining a random sample of size

*n* = 25 with a sample mean $\overbar{x} $of 48 or less.

1. Is the observed sample mean $\overbar{x}=48$ statistically significant? Explain.
2. State a conclusion about the assumed value of the population mean.
3. What can we conclude about the mean highway gas mileage for all of the manufacturer’s hybrid cars?