**Unit 5: Investigation 7 (2 Days)**

**Parabolas**

**Common Core State Standards**

* G-GPE.2. Derive the equation of a parabola given a focus and directrix.

**Overview**

Overall, in this investigation, students discover that a parabola is a locus of points that are equidistant from a fixed line (directrix) and a fixed point (focus). Students begin this investigation by recalling the definition of circle as a locus of points that are equidistant from a fixed point. With this prior understanding of locus, using dynamic geometry software students then explore the locus of points that are equidistant from a fixed line and point. With this new definition of a parabola students will then derive the equation of a parabola given a focus and directrix. Once the general equations have been derived, students will then explore both skills and real world application problems.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Derive the standard form equations for the parabola with a horizontal directrix and the parabola with a vertical directrix.
* Given the equation of a parabola, determine both the focus and the directrix.
* Given two of the following, determine the equation of the parabola: vertex, focus, or directrix
* Solve real world applications involving parabolas.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 5.7.1** requires students to determine the vertex, focus, and directrix of a parabola given its equation.
* **Exit Slip 5.7.2** requires students to solve a real-world problem involving a parabola.
* **Journal Entry** asks students to explain the locus definition of parabola.

**Launch Notes**

Begin this investigation by referring back to the definition of a circle. In reviewing the circle, help students to understand the concept behind a locus. It is a set of points that satisfy a specific conditions or criteria. Every point that lies on a circle is the same distance from the center, and any point in the plane that distant from the center lies on the circle. You may also have them refer back to another familiar locus: the set of all points that are equidistant from the endpoints of a segment. As we saw in Investigation 2 of this unit, this locus is the perpendicular bisector of the segment.

Once your students recall and understand locus then they are ready for **Activity 5.7.1.**

**Teaching Strategies**

In **Activity 5.7.1** **Exploring the Parabola as a Locus of Points** students begin by trying to imagine a locus that has the condition; all points are equidistant from both a line and a point. Have them draw a line and a point *A*, not on the line. Then instruct them to try to find points that are equidistant from both the line and point *A*. This prediction phase is intended to help students develop a deeper understanding of locus. After students have time to make predications and share them with the class they will test their predictions by exploring this locus using paper folding (**Activity 5.7.1a**) or a dynamic geometry software program (**Activity 5.7.1b**). In the GeoGebra version students are given a premade file in which they grab a point that is equidistant from the focus (point) and directrix (line). As they move the point around the plane it remains equidistant to the focus and directrix but leaves a trace or trail of points in its path. This trace reveals that the locus is a parabola. The paper folding method should generate enough points to create the same effect.

**Differentiated Instruction (Enrichment)**

Instead of giving students the pre-made file in Activity 5.7.1b, challenge them to use the dynamic geometry software to create a file that can be used to investigate this locus.

**Activity 5.7.2 Parabolas in the Coordinate Plane** provides another way to observe the parabola as the locus of points equidistant from the focus and the directrix. It also introduces students to the family of functions *f*(*x*) = *ax*2 for parabolas with vertex at the origin and line of symmetry on the y-axis. Changing the value of *a* enables students to experiment to find the equation of the parabola with focus at (0,1) and directrix: *y* = – 1.

There are two versions. Activity 5.7.2a does not require technology. Activity 5.7.2b uses GeoGebra.

In **Activity 5.7.3** **The Equation of a Parabola in Standard Form** students will use the locus definition of a parabola to derive the standard form of the equation of a parabola with directrix at *y* = – *p* and focus at (0, *p*). Before generalizing, the derive the equation for the special case where p =1. They also verify that the *y-*axis is a line of symmetry for a parabola in standard position.

**Differentiated Instruction (For Learners Needing More Help)**

The algebraic derivation of the equation of a parabola may be daunting for many students. You may want to skip this or have them only do the special case in questions 1–6 of **Activity 5.7.3**.

You may want to point out to students that in algebra courses parabolas are usually associated with quadratic functions. Thus the parabola in standard position has the equation *f*(*x*) = *ax*2. When we view the parabola as a locus however, we derive the equation $x^{2}=4py.$ We can solve for *y* to get $y=\frac{1}{4p}x^{2}$. When *p* = 1 we have$ y=\frac{1}{4}x^{2}$.

**Group Activity**

Have students work in groups of four. Give each group four values of *p*, two positive and two negative, and assign one value to each group member. Students draw the graph of $x^{2}=4py$ for their particular value of *p*, labeling the focus, the vertex, and the directrix. They then look at their colleagues’ graphs and use them to figure out the value of *p* that was used.

**Exit Slip 5.7.1** may be given following **Activity 5.7.3**.

In **Activity 5.7.4 Applications of Parabolas** students continue to practice skills involving parabolas but move more to solving real world situations involving parabolas. Some of these situations include parabolic satellite dishes, lenses, headlights, etc. All of these problems may be solved by placing the parabola on a coordinate grid with the vertex at the origin and focus on the *y-*axis. Then use the standard form of the equation, $x^{2}=4py.$

**Exit Slip 5.7.2** may be given any time after **Activity 5.7.4.**

**Activity 5.7.5 Translating Parabolas** introduces parabolas that are not in standard position. When the parabola $x^{2}=4py$ is translated by the vector [*h*, *k*] the equation of the translated parabola may be written $(x-h)^{2}=4p\left(y-k\right).$ You may want to compare this equation with the general equation for the circle $(x-h)^{2}+(y-k)^{2}=r^{2}$, developed in Investigation 1. For the parabola (*h*, *k*) is the translated vertex; for the circle it is the translated center. This investigation also introduces a parabola with vertex at the origin rotated 90° from standard position. Rotations through any angle may be studied in a more advanced course.

**Journal Entry**

How are the focus and directrix of a parabola related to the points on the parabola? Look for students to refer to the locus definition.

**Closure Notes**

Have students create their own locus definitions. For example, one student might say, find the locus of points that are twice as close to one endpoint of a segment as they are to the other endpoint. Students should be encouraged to use a dynamic geometry software to explore both their own and their classmate’s locus.

**Vocabulary**

directrix (of parabola)

focus (of parabola)

locus

parabola

vertex (of parabola)

**Coordinate Geometry Equation**

**Parabola with focus (0, *p*) and directrix *y* = – *p*:** $x^{2}=4py.$

**Resources and Materials**

Parchment paper, rulers for Activity 5.7.1a

GeoGebra for Activities 5.7.1b and 5.7.2b

ctcoregeomACT571.ggb

ctcoregeomACT572.ggb

Activities:

 Activity 5.7.1a Exploring the Parabola as a Locus of Points (Paper Folding)

 Activity 5.7.1b Deriving the Equation of a Parabola in Standard Form (GeoGebra)

 Activity 5.7.2a Parabolas in the Coordinate Plane (without technology)

 Activity 5.7.2b Parabolas in the Coordinate Plane (GeoGebra)

 Activity 5.7.3 The Equation of a Parabola in Standard Form

 Activity 5.7.4 Applications of Parabolas

 Activity 5.7.5 Translating Parabolas