**Unit 6: Investigation 5 (2 Days)**

**Spheres**

***CCSS:***

G-GMD.2. (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.

G-GMD.3.Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

**Overview**

After discovering formulas for the surface area and volume of a sphere through experimentation, students see the classical proof that a volume of a hemisphere is the difference between the volume of a cylinder and that of a cone with height = *r*. Then a dissection argument shows that the sphere may be considered made up of many cones; thus leading to the conclusion that its surface are is 4π*r2*. The investigation concludes with a number of applications.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Explore informally various ways to demonstrate the surface area and volume of spheres.
* Understand a proof for the volume formula of a sphere using Cavalieri’s principle.
* Use the formulas for surface area and volume in applications involving spheres.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 6.5** involves applying the formulas for volume and surface area to two planets.
* **Journal Entry** asks students to explain the derivation of the volume and surface area formulas.

**Launch Notes**

Recall the experiments we did with the transparent shapes using colored water and oranges to initiate the unit. What do we think are the formulas for area and volume of a sphere? In this investigation we’ll gain a deeper understanding of why they work.

**Teaching Strategies**

**Activity 6.5.1 Surface Area and Volume of a Sphere** has students perform two experiments that lend support to the conjectures they made earlier. The surface area of a sphere is modeled by the skin of an orange which can be peeled and arranged to fill 4 circles with the same radius as the sphere. The volume is found by cutting the sphere into conical wedges. Since the volume of a cone is $\frac{1}{3}$ times the area of its base times its height, the sum of all the cone volumes is $\frac{1}{3}$ times the sphere’s surface area times its radius.

**Differentiated Instruction (For Learners Needing More Help)**

Students should have an opportunity to repeat the experiments with the transparent sphere and cone and orange for themselves, in conjunction with Activity 6.5.1

**Activity 6.5.2 Cavalieri’s Principle and the Volume of a Sphere** presents the proof, using Cavalieri’s principle that the volume of a hemisphere is $\frac{2}{3}$ times the volume of a cylinder with radius and height both = *r*. This is preceded by a GeoGebra demonstration that helps students see that the area of a cross section of the cone of the same radius and height is equal to the area of the ring that represents the cylinder minus the sphere. Then several arguments are presented to show that the surface area is 4π*r2*.

**Differentiated Instruction (Enrichment)**

More able students can prepare a PowerPoint presentation based on researching how different individuals through history have come to develop the formulas for volume and surface area of the sphere.

Introduce Activity 6.5.3 by discussing the possibility of terraforming Mars into Earth like planet. You may wish to use this video: <https://www.youtube.com/watch?v=9F1iWp4Gl3k>, which runs just under 5 minutes.

**Activity 6.5.3 Spherical Applications** this gives students an opportunity to solve basic problems involving volumes and areas of spheres. The first four questions focus on the problem of making the planet Mars inhabitable for humans. The problems focus on fundamental issues, like how much water or vegetation is needed to be developed. The students use the formulas for the area and volume of the sphere to estimate the volume of water on Earth, the size of Mars' core or how many years would be necessary to develop a basic vegetation on Mars. The problem on soap bubbles introduces that fact that the sphere is the shape that minimizes surface area for a given volume. The soccer ball problem introduces additional variable such as air pressure and density. Students may want to apply dimensional analysis in approaching these questions.

**Closure Notes**

This Investigation completes the development of the basic formulas for surface area and volume for geometry and begins to anticipate the methods that will be used in a calculus course to compute volumes of more complex solids.

**Group Activity**

Students may play a matching game to help learn the formulas for surface area and volume and the properties of solids. Use the template provided. Cut 32 cards from each template. Shuffle the cards and give five to each player and place the remaining cards in a stack. One each turn the player draws a card. They lay out a pair that match (e.g “prism” and *V* = *Bh* or “cone” and “has an apex”) They may also play a card to match one that is already on the table. If they can’t play, they discard one card and draw another card. First player to play all their cards wins.

**Journal Entry**

Explain to a student who missed class, how we were able to find formulas for the volume and surface area of a sphere. Look for students to explain how these two formulas are related to each other.

**Vocabulary**

hemisphere

**Theorem**

**Hemisphere-Cylinder Theorem:** The volume of a hemisphere is equal to $\frac{2}{3}$ the volume of a cylinder with the same radius and height equal to its radius.

**Resources and Materials**

Oranges, paper, pencil, play dough, floss for Activity 6.5.1

GeoGebra program: ctcoregeomACT652 for Activity 6.5.2

Template for Group Activity: Unit\_6\_Investigation\_5\_matching\_game.docx

Video

[www.youtube.com/watch?v=6EzQEdBX\_30](http://www.youtube.com/watch?v=6EzQEdBX_30). (Alternative proof for surface area in Activity 6.5.2)

<https://www.youtube.com/watch?v=9F1iWp4Gl3k> (to introduce Activity 6.5.3)

[www.youtube.com/watch?v=iQigGLdSsUo](http://www.youtube.com/watch?v=iQigGLdSsUo) (Soap bubbles for question 9, Activity 6.5.3)

Earth facts:

<http://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html>

Mars facts:

<http://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html>

Lichens

<http://www.earthlife.net/lichens/growth.html>

Mars Core

<http://www.geo.umass.edu/courses/geo892/archive/stevenson_mars_mag.pdf>

Ice Caps melting

<http://science.howstuffworks.com/environmental/earth/geophysics/question473.htm>

Christensen, P. R. (2006). "Water at the Poles and in Permafrost Regions of Mars".*GeoScienceWorld* Elements **3** (2): 151–155.