**Activity 7.2.4 The Fundamental Counting Principle**

Recall that if a random process has a sample space *S* in which each outcome is equally likely, then the probability of an event *A* is



In this activity, we focus on counting methods that can help us calculate the number of outcomes in a sample space or an event.

1. Suppose a sandwich shop opens and gives the following choices for sandwiches:

* Select bread: wheat, white, sub roll
* Select meat: turkey, ham

a. Draw a tree diagram showing all possible choices for sandwiches. How many choices are there?

b. In order to entice more customers, the shop adds roast beef and salami to the meat choices. Now how many sandwich choices are there? Explain how you got your answer.

c. Customers asked for more bread choices. The sandwich shop responded by adding wraps and seven-grain bread to its menu. With the 5 choices of bread and 4 choices for meats, how many different sandwiches can be made? Explain how you got your answer.

d. After surveying its customers, the sandwich shop adds the possibility of cheese to the menu and allows customers to select no cheese, American cheese or Swiss cheese. Now how many different sandwiches are possible? Explain how you got your answer.

e. Suppose that the sandwich shop makes one of each type of sandwich. You randomly select one of the sandwiches. What is the probability that your sandwich is one of the sandwiches with cheese? Explain how you got your answer.

Question 1 provides an example of what is called the Fundamental Counting Principle.

**Fundamental Counting Principle**

Given a sequence of two choices, suppose there are *m* ways of choosing the first thing and *n* ways of choosing the second after the first has been chosen. Then the total number of choice sequences is *m* × *n*.

2. As demonstrated by your work in question 1, the Fundamental Counting Principle can be extended to a sequence of three choices. Express the Fundamental Counting Principle so that it applies to a sequence of three choices.

3. Consider a well-shuffled standard deck consisting of 52 cards. Caleb is asked to draw a card from the deck, record his result in the space below, then replace the card back in the deck and reshuffle. He then repeats this process until five cards have been drawn from the deck and recorded in the spaces below. Because of the replacement, it is possible for Caleb to draw the same card more than once. Caleb does not rearrange the cards he has picked.

Card #5

Card #4

Card #3

Card #2

Card #1

a. How many possible outcomes are there for Caleb?

b. Of those possible outcomes, how many outcomes have Caleb drawing all hearts?

c. What is the probability that Caleb drew all Hearts?

4. Consider a well-shuffled standard deck consisting of 52 cards. Juan is asked to draw a card from the deck, record his result in the space below, but NOT replace the card he drew. He then repeats this process until five cards have been drawn from the deck. Because cards are not replaced after drawing, it is not possible for Juan to draw the same card more than once. Juan does not rearrange the cards he picked.

Card #5

Card #4

Card #3

Card #2

Card #1

a. How many possible outcomes are there for Juan?

b. Of those possible outcomes, how many outcomes have Juan drawing all hearts?

c. What is the probability that Juan had all Hearts?

5. A jar contains six jellybeans, one red, one yellow, one green, one black, one pink, and one blue.

a. If all of the jellybeans are removed from the jar and placed in a row, how many different arrangements are possible?

b. If five of the jellybeans are randomly chosen and placed in a row, how many different arrangements are possible?

c. If four of the jellybeans are randomly chosen and placed in a row, how many different arrangements are possible?

d. If three of the jellybeans are randomly chosen and placed in a row, how many different arrangements are possible?

e. If two of the jellybeans are randomly chosen and placed in a row, how many different arrangements are possible?

6. Suppose the jellybeans in question 5 are removed from the jar and placed into a paper cup. In this case, you are not arranging the jellybeans in any order, but instead are just considering the collection of jellybeans that get put into your cup.

a. How many different cups of jellybeans are possible if two jellybeans are removed from the jar and placed into the cup? Keep in mind that putting a green jellybean into the cup followed by a yellow jellybean gives the same results as putting a yellow jellybean into the cup followed by a green jellybean. In both cases, the cup contains one yellow and one green jellybean.

b. How many different cups of jellybeans are possible if six jellybeans are removed from the jar and placed into the cup?