**Activity 7.2.5 Permutations and Combinations**

When we determine probabilities of events for situations in which the outcomes in the sample space are equally likely, it helps to be able to count the number of outcomes both in the sample space and in the event. In order to do this, we must think about certain aspects of the events. Here are two important questions:

* Does order matter in the event?
* Do previous outcomes affect future outcomes?

A review of the counting techniques covered so far in Investigation 2 can be found on the last page of this activity.

1. Your teacher is arranging 20 students in a line in the front of the room for a class picture and you are interested in counting the number of possible pictures she could take (based upon the number of arrangements of students).

a. Does the order of the students matter when counting the possible number of pictures? Explain.

b. If your teacher chooses a student to stand in the first position in the line, does that have an effect on how many students she can choose from for the second position?

c. How many different arrangements for the picture are there? Is this an example of permutations or combinations?

d. Sharon is hoping she will be first in line. If the teacher randomly chooses the order in which she arranges the students in line, what is the probability that Sharon will be first in line?

2. Brianna, Daniel, and Avery are among twenty finalists for a chance to win a free cruise. Three identical cruises will be awarded.

a. Does the order of the finalists matter in awarding the three cruises? Explain.

b. How many different outcomes are there for selecting three of the finalists for the cruises? Is this an example of permutations or combinations?

c. What is the probability that Brianna, Daniel, and Avery are the ones chosen?

3. To play the Connecticut Lottery’s Mega Millions game, you begin by choosing 5 different “white” numbers from 1 to 75 (the order does not matter). Then you choose one Mega Ball “yellow” number from 1 to 15.

a. How many possible ways are there to choose the 5 “white” numbers (not including the Mega Ball)? Is this an example of combinations or permutations?

b. How many possible ways are there to play the Mega Millions game? In other words, how many ways are there to choose the 5 “white” numbers and the Mega Ball “yellow” number? (Hint: You’ll need the Fundamental Counting Principle.)

c. What is the probability of winning the Mega Millions jackpot? To win you must match all 5 of the “white” numbers and the Mega Ball “yellow” number.

d. The prize for matching all 5 of the “white” numbers but not the Mega Ball “yellow” number is $1,000,000. What is the probability of winning the million dollar prize?

4. John forgot the combination to the lock on his locker (see below). His combination is a sequence of three numbers 0-39 where any number can occur more than once.

a. Does the order of the three numbers matter on combination locks?

b. Does the previous outcome in the sequence of numbers affect the next outcome?

c. How many possible outcomes are there on the first spin? The second spin? The third spin?

d. If John wanted to try all the possible combinations, how many would he have to try?

e. What is the probability that John guesses the right combination on his first try and is able to open his lock?

5. Suppose you have the following four Scrabble® tiles: A, C, T, S. 

a. How many three letter sequences you can make from the four tiles?

b. Make a list of all possible three-letter sequences that can be made from the letters A, C, T, S. check that you have written the same number of sequences as your answer to (a).

c. Suppose the tiles are placed in a container and without looking you draw out a three-letter sequence. What is the probability that you get an actual word? (If you are not sure if you found all the words, go to <http://www.merriam-webster.com/scrabble>. Enter the four letters A,C,T,S and click on the search symbol.)

6. Suppose you have 12 books in a stack.

a. You only have space for 6 books on your bookshelf. In how many different ways can you select 6 books from the stack and arrange them in the empty space on your bookshelf? Is this an example of combinations or permutations?

b. You want to bring some of the books into school but you only have room in your backpack for 6 books. How many different selections of 6 books can you make? Is this an example of combinations or permutations?

**Review of Counting Techniques**

**Fundamental Principle of Counting**

 If there are *n*1 ways of choosing one thing, *n*2 ways of choosing a second after the first is chosen, . . . , and *nk* ways of choosing the last item after the earlier choices, then the total number of choices for the sequence is $n\_{1}×n\_{2}×\cdots n\_{k}$

A **permutation** is an ordered arrangement of *k* items that are chosen without replacement from *n* items. The number of arrangements, denoted as , can be calculated using the following formula: *nPk =*$n\left(n-1\right)\left(n-2\right)\cdots \left(n-k+1\right)=\frac{n!}{\left(n-k\right)!}$

where $n!=n(n-1)(n-2)\cdots 2∙1$ and 0! = 1.

A **combination** is an unordered selection of *k* items that are chosen without replacement from *n* items. The number of combinations, denoted as , can be calculated from the following formula: 