**Unit 7 Probability**

**(4–5 Weeks)**

**UNIT PLAN**

This unit on probability builds on background students bring to this course. Ideally, students have had some experience with probability in the Grade 7 and 8 Core Curriculum. In practice, we realize that probability may be a new topic for many students and teachers. However, we expect students to understand that probability measures the likelihood that a chance event occurs.

**Investigation 1 Sample Spaces** introduces the concept of random processes and sample spaces, the set of all possible outcomes of a random process. Students define sample spaces and events using set notation. They form new events from unions, intersections, and complements of other events. Working with area models for probability, students discover two rules of probability, the General Addition Rule and the Complement Rule.

**Investigation 2 Theoretical and Experimental Probability** introduces two approaches to assigning probabilities. In the first approach (theoretical), sample spaces must have a finite number of equally likely outcomes. The probability that an event *A* occurs is computed by dividing the number of individual outcomes in *A* by the number of outcomes in the sample space, *S*. Counting methods—the Fundamental Counting Principle, permutations, and combinations—are developed to aid in counting the number of outcomes in events and sample spaces. The second approach (experimental) requires computation of relative frequencies based on many, many trials of a random process.

**Investigation 3 Independent Events and the Multiplication Rule** introduces the concept of independent and dependent events. Students use the Multiplication Rule for Independent Events in two ways: (1) to calculate the probability that two independent events both occur and (2) to check whether or not two events are independent given information about their probabilities. This investigation concludes with an informal discussion of conditional probabilities from everyday situations, which sets the stage for Investigation 4.

**Investigation 4** **Conditional Probability** developsthe concept of conditional probability more formally. In cases where outcomes in the sample space are equally likely or events are represented by area probability models, students calculate the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*. The general formula for is presented and used to derive the General Multiplication Rule: .

**Investigation 5 Interpreting Two-Way Frequency Tables** introduces the technique of organizing bivariate categorical data in two-way tables. In order to analyze data from a two-way table, students learn to calculate marginal, joint, and conditional relative frequencies/percentages. If the data set is large and representative of the population, these relative frequencies can be used to estimate probabilities. To help visualize patterns in the data, students represent marginal and conditional percentages with bar graphs. Using tables of percentages and/or bar graphs, students interpret the percentages in the context of the data and identify associations and trends.

**Investigation 6 Using Probability to Make Decisions** introduces the concept ofexpected value in the context of playing games of chance. Students then apply what they have learned about expected value to the contexts of whether or not to purchase insurance or to guess on a multiple choice test. The need to randomly select answers to multiple choice questions leads to use of a calculator’s random number generator to choose the answer. The final activity focuses on the imperfect nature of drug and medical tests. Given information about a test for some trait (medical condition, drug use, etc.) and the prevalence of the trait in the population, students determine the probability that a person has the trait given a positive test result.

The **Performance Task** for this unit, the Happiness Survey, asks students to conduct a survey. The first two items on the survey ask respondents to rate the physical beauty of their school and to rate their level of happiness on the day of the survey; students write two or three additional questions of their own. They design a sampling plan, administer the survey according to their sampling plan, organize the data into two-way tables, calculate appropriate percentages, create appropriate graphic displays and interpret the results. Using percentages as estimates of probability, students also answer questions about independence among responses to the questions.

**Essential Questions**

* What is probability?
* How are events defined?
* What is meant by independent/dependent events?
* How are probabilities, including compound probabilities, calculated?
* What is conditional probability and how does conditional probability apply to real-life events?
* How are two-way frequency tables used to model real-life data?
* What is expected value?

**Enduring Understandings**

* Understand that it is not possible to predict with certainty short-term behavior of random phenomena but it is possible to use probability to predict long-run patterns.
* Probability models are useful tools for making decisions, choices, or predictions.

**Unit Contents**

Investigation 1 Sample Spaces (4 days)

Investigation 2 Theoretical and Experimental Probability (4 days)

Investigation 3 Independent Events and the Multiplication Rule (3 days)

Investigation 4 Conditional Probability (2-3 days)

Investigation 5 Interpreting Two-Way Frequency Tables (3 days)

Investigation 6 Using Probability to Make Decisions (3 days)
Performance Task (2-3 days)

Review (1 day)

End of Unit Test (2 days)

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. **Make sense of problems and persevere in solving them.**

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. **Model with mathematics.**

5. Use appropriate tools strategically.

6. Attend to precision.

7. **Look for and make use of structure.**

8. Look for and express regularity in repeated reasoning.

**Content Standards Overview**

* Summarize, represent, and interpret data on two categorical and quantitative variables.
* Use the rules of probability to compute probabilities of compound events in a uniform probability model.
* Understand independence and conditional probability and use them to interpret data.
* Use probability to evaluate outcomes of decisions.

**Content Standards**

* S-ID-5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies) Recognize possible associations and trends in the data.
* S-CP-1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
* S-CP-2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
* S-CP-3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
* S-CP-4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
* S-CP-5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
* S-CP-6 Find the conditional probability of *A* given *B* as a fraction of *B*’s outcomes that also belongs to A, and interpret the answer in terms of the model.
* S-CP-7. Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.
* S-CP 9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
* S-MD-5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.

b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

* S-MD 6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
* S-MD 7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

**Laws, Rules, and Formulas**

Investigation 1

Given events *A* and *B*:

 **De Morgan’s laws:**  and .

 **General Addition Rule:** Given events *A* and *B*, **Complement Rule:** Given event *A*, .

Investigation 2

Given a sample space *S* in which individual outcomes are equally likely and an event *A*,

**Fundamental Counting Principle:** If there are ways of choosing one thing,ways of choosing a second after the first is chosen, . . ., and ways of choosing the last item after the earlier choices, then the total number of choices for the sequence is$ n\_{1}×n\_{2}×\cdots ×n\_{k}$.

**Permutations:** The number of arrangements k distinct items that are chosen without replacement from a collection of n distinct items is

*nPk* $=n×\left(n-1\right)×\cdots ×\left(n-k+1\right)=\frac{n!}{\left(n-k\right)!}$

**Combinations:** The number of ways of choosing *k* distinct items from *n* distinct items where order is not important is

*nCk*$=\frac{nPk}{k!}=\frac{n×\left(n-1\right)×\cdots ×\left(n-k+1\right)}{k!}= \frac{n!}{k!\left(n-k\right)!}$

Investigation 3

**Multiplication Rule for Independent Events:** If *A* and *B* are independent events, then . Moreover, if for any events *A* and *B*, , then *A* and *B* are independent.

Investigation 4

**Conditional Probability, Case of Equally Likely Outcomes:** Given events *A* and *B*, provided the number of outcomes in *B* is positive.

**Conditional probability:** Given events A and B with , .

**General Multiplication Rule:** Given events *A* and *B*, provided *P*(*B*) > 0.

Investigation 5

Given a two-way table:

**Joint percentages** of two variables: [(Cell entry)/(Grand Total) × 100]%.

**Marginal percentages** for one variable: [(Total entry)/(Grand Total) × 100]%.

**Conditional percentages:**

* To find the conditional percentages for the row variable for each level of the column variable, compute the **column percentages**: [(cell entry)/(column total) × 100]%.
* To find conditional percentages for the column variable for each level of the row variable, compute the **row percentages**. [(cell entry)/(row total) × 100]%.

Investigation 6

**Expected Value:** Given a finite probability model

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Outcome |  |  | ... |  |
| Probability |  |  | ... |  |

Expected value = *x*1*p*1 + *x*2*p*2+ ... + *xnp*

**Bayes Rule:** Bayes Rule provides a connection between *P*(*A*|*B*) and *P*(*B*|*A*).
(Students calculate Bayes Rule problems without use of this formula.)
, provided and .

**Vocabulary**

area probability model

Bayes Rule

column variable

combination

conditional probability

conditional relative frequencies/percentages

dependent events

event

expected value

false negative

false positive

fundamental counting principle

general multiplication rule

grand total

joint relative frequencies/percentages

independent events

intersection of two events

marginal relative frequencies/percentages

multiplication rule for independent events

mutually exclusive events

permutation

probability

probability model

random process

relative frequency of an event

row variable

sample space

true negative

true positive

two-way table

union of two events

variable

**Assessment Strategies**

**Performance Task**

Students will analyze survey data using techniques learned from their work with two-way tables. Students compute relative frequencies as estimates of probabilities and apply some of their knowledge about probability to assess the likelihood that certain events occur and whether or not two events are independent.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* End of Unit test

**Alternative Path Through Unit 7 (4 weeks)**

Although it would be best to cover Unit 7 in its entirety, that may not be possible. Below is an outline of one possible shortened path through this unit.

Investigation 1: (3 days)

 Activity 7.1.1

 Activity 7.1.2

 Assign Activity 7.1.4 for homework

 Cover in class questions 1 and 2 of Activity 7.1.5, verifying De Morgan’s Laws

 Activity 7.1.6

 Assign Exit Slips 7.1.1 and 7.1.2 as homework.

Investigation 2: (2 days)

 Activity 7.2.1

 Assign Activity 7.2.3 for homework

 Exit Slip 7.2.1

 Coverage of counting techniques (If really short on time, skip this topic):

 Activity 7.2.4

 Assign Activity 7.2.5 for homework. (Or minimize the coverage of counting techniques and skip this activity)

 Exit Slip 7.2.2

Investigation 3: (2 days)

 Activity 7.3.1

 Assign 7.3.2 for homework.

 Exit Slip 7.3.1

 Activity 7.3.3

 Assign Exit Slip 7.3.2 and Journal Entry for homework.

Investigation 4: (2 days)

 Activity 7.4.1

 Assign Activity 7.4.2 for homework.

 Exit Slip 7.4.1

 Activity 7.4.3

 Assign Journal Entry and Exit Slip 7.4.2 for homework.

Investigation 5: (3 days)

 Activity 7.5.1

 Assign video: Unit 13, Two-Way Tables from *Against All Odds* for homework

 ([www.learner.org/courses/againstallodds](http://www.learner.org/courses/againstallodds)) along with Journal Entry.

 Exit Slip 7.5.1

 If students need to review the material from Activity 7.5.1, assign Activity 7.5.2 for

 homework. Assign the Journal Entry for homework.

 Activity 7.5.3

 Exit Slip 7.5.2. Time remaining, begin work on Performance Task. Groups should write two questions for the survey questionnaire and discuss a sampling plan for administering the survey. If running short on time, instructor should select two questions from those submitted by the groups and a reasonable sampling plan. Otherwise, class can select the questions and sampling plan. Outside of class, students (or instructor) oversees the administration of the survey and enters the data into a spreadsheet.

Investigation 6: (1 day)

 Activity 7.6.1

 Assign Exit Slip 7.6.1 and Journal Entry for homework.

Performance Task: (2 days)

Groups work on performance task and analyze the Happiness Survey data.

End of Unit Assessment: (1 day) Omit any questions that were not covered.