**Activity 6.1.8 Convert Between Degrees and Radians**

In previous activities, you sketched an arc of a given length and the subtended central angle. You noticed that one revolution = 360° = 2π radians. By taking fractions of a circle, you identified both the radian measure and the degree measure for many central angles in standard position and the length of the arc subtended by that angle. How do you determine radian and degree measures when the arc is not a convenient fraction of the circle? In this activity, you will learn an algebraic process for changing back and forth between radians and degrees.

Using conversion factors to convert from one unit of measure to another:

1. One method for converting from one unit of measure to another is to use a conversion factor.

a. Example, you know that 1 hour = 60 minutes, so 1 hr/60 min = \_\_\_\_\_\_\_\_\_\_\_.

Write 150 minutes in terms of hours by multiplying 150 minutes by the conversion factor 1 hr/60 min. Show your work here. Be sure to write the units of measure.

b. Convert 5.25 feet to inches using the conversion factor method. (You will have to figure out the conversion factor based on what you know about feet and inches.)

 c. Since 180$°$ = π radians, what is 180$°$/ π radians? \_\_\_\_

d. Write the following radian measure in terms of degrees by multiplying by $\frac{180°}{π radians}$.

 $\frac{π}{3}radians$

e. Write the following degree measure in terms of radians by multiplying by $\frac{π}{180°}$ . Do not use decimals. Rather, factor the numerators and denominators and simplify the fraction.

 330$°$

f. At 1 o’clock, what angle is formed by the large hand and the small hand of a clock?

* in degrees? \_\_\_\_\_\_
* in radians?\_\_\_\_\_\_\_

g. To convert radians to degrees, multiply by: \_\_\_\_\_\_\_\_

h. To convert degrees to radians, multiply by:\_\_\_\_\_\_\_\_

2. Do the following conversions and fill in the blank. Then sketch the designated angle in standard position on a coordinate plane. Use simplified fractions, not decimals.

a. $\frac{7π}{6} radians= \\_\\_\\_\\_\\_\\_\\_degr$ees b. $-\frac{9π}{4} radians= \\_\\_\\_\\_\\_\\_\\_degr$ees

c. 135° =\_\_\_\_\_\_ radians d. -270° = \_\_\_\_\_\_\_\_\_ radians

d. 2π radians= \_\_\_\_\_\_\_\_\_° e. 0° = \_\_\_\_\_\_\_\_\_\_\_radians

When describing the measurement of something (length, volume, temperature, angle measure…), it is customary to write the unit of measure EXCEPT for radians. An angle measure of $\frac{5π}{3}$ is understood to be in radians, whereas an angle of $\frac{5π}{3}$ ° is measured in degrees because it is so marked. Feel free to write in the word “radians” or an abbreviation like “rad” when working with radians, if it helps.

3. Convert between radians and degrees. You may use decimal approximations when doing the following conversions, because they are not the special angles formed by dividing a circle in eights or twelfths. Remember that if there is no unit of measure, we mean “radian”. Round decimals to the nearest 100th .

1. 100$°$ ≈\_\_\_\_\_\_\_\_\_ radians
2. $\frac{3π}{5}= \\_\\_\\_\\_\\_\\_\\_°$
3. 314$° $≈\_\_\_\_\_\_\_\_\_ radians
4. $1°$ ≈ \_\_\_\_\_\_\_\_\_ radians
5. 1 ≈\_\_\_\_\_\_\_\_ degrees
6. 2 ≈\_\_\_\_\_\_\_\_degrees
7. 2π =\_\_\_\_\_\_ degrees

4. i. Do the following conversions and fill in the blanks. Use simplified fractions, not decimals.

ii. Sketch the designated angle in standard position on a coordinate plane.

iii. Write an angle between 0° and 360° and between 0 and 2π that is co-terminal with the given angle. “Co-terminal” means the angles have the same terminal ray. You can add or subtract as many complete revolutions as needed to find a co-terminal angle.

iv. On the graph label the measure of the **acute angle** that is defined by the terminal ray of the angle and the x axis.

Example:

 **i.** $\frac{15π}{4}$ **= \_675\_°**

$\frac{15π radians}{4} \left(\frac{180°}{π radians}\right)= \frac{15 }{2} \left(\frac{90°}{1}\right)=\frac{15 }{1} \left(\frac{45°}{1}\right)= 675°$

**Co-terminal angle in degrees:**

675 – 360= 315 (one rotation)

315° is an angle co-terminal with $675°$

Note: 315-360= -45, showing that 315° is 45° short of 360°

**ii Sketch**

675°

45°

**iii.** $\frac{15π}{4}$ **is co-terminal with an angle of measure** $\frac{7π}{4}$ **or 315°**

Co-terminal angle in radians:

Subtract $2π from \frac{15π}{4} by rewriting 2π as\frac{8π}{4}$:

$ $

$\frac{15π}{4}-\frac{8π}{4}=\frac{7π}{4} $

$\frac{7π}{4} $ is co-terminal with $\frac{15π}{4}$ . Note $\frac{7π}{4}-2π=-\frac{π}{4} $ showing that $\frac{7π}{4} $is $\frac{π}{4}$ short of 2π.

**iv. On the graph, label the measurement of the acute angle formed with x axis.** (the example shows the -45° angle indicated on the graph)

5a.

i. $ \frac{17π}{3}$ is coterminal with with an angle of measure \_\_\_\_\_radians or \_\_\_\_\_ degrees.

ii Sketch angle in standard position.

 iii. On the graph, label the measurement of the acute angle formed formed by the terminal ray of the angle and the x-axis.

5b.

i. $\frac{13π}{4}$ is co-terminal with with an angle of measure \_\_\_\_\_\_\_ radians or \_\_\_\_\_\_\_\_degrees .

ii Sketch angle in standard position.

 iii. On the graph, label the measurement of the acute angle formed formed by the terminal ray of the angle and the x-axis.

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5c.

i. $-480°$ is co-terminal with with an angle of measure \_\_\_\_\_\_\_\_\_ degrees or \_\_\_\_ radians

ii Sketch angle in standard position.

 iii. On the graph, label the measurement of the acute angle formed formed by the terminal ray of the angle and the x-axis.

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5d.

 i. $-\frac{13π}{6} $is co-terminal with with an angle of measure \_\_\_\_\_\_\_ radians or\_\_\_\_\_\_\_\_degrees .

ii Sketch angle in standard position.

 iii. On the graph, label the measurement of the acute angle formed formed by the terminal ray of the angle and the x-axis.