**Activity 6.6.1 Doing Algebra with Trigonometric Functions**

Operating with Trigonometric Functions is much the same as operating with any algebraic expression. With trigonometry, you can also simplify further using the Pythagorean Identity

**sin2(*x*) + cos2(*x*) = 1.**

Solve as indicated:

1. a. Solve for x2: x2 + y2 = 1 b. Solve for x: x2 + y2 = 1

c. Solve for sin2(*x*): sin2(*x*) + cos2(*x*) = 1 d. Solve for sin(*x*): sin2(*x*) + cos2(*x*) = 1

e. Solve for cos2(*x*): sin2(*x*) + cos2(*x*) = 1. f. Solve for cos(*x*): sin2(*x*) + cos2(*x*) = 1.

NOTE: The equations you obtained in parts 1c-1d are useful to recognize. These are new identities, that means that the equations are true for all real numbers. Use them along with the identity ‘sin2(*x*) + cos2(*x*) = 1’ to rewrite trigonometric expressions.

Simplify:

2a. Multiply: (1+ x)(1– x) (1+sin(*x*))(1 – sin*(x*))

b. (x + 3)2  (sin(*x*)) + cos(*x*))2

(sin(*x*) – cos(*x*))2

c. Does sin2(*x*) + cos2(*x*) equal (sin(*x*)+cos(*x*))2 ? Explain in words and symbols.

d. Try to answer quickly, just by observation: What is sin23 + cos23? \_\_\_\_\_

3 a. Factor: x3 +xy2 (sin3(*x*)) + sin(*x*) ∙(cos2(*x*))

Factor:

b. x2 – y2 cos2(*x*) – sin2(*x*)

c. 4x2 – 9y2 4 – 9sin2(*x*)

d. 1 – x4 factor, then simplify using a trigonometric identity: 1 – cos4(*x*)

4. On the unit circle the x coordinate of the point W(t) is the cos(t), the y coordinate is sin(t) and the ratio of the coordinates $\frac{y}{x}$ is tan(t).

This gives us another useful identity:

$$tan⁡(t)= \frac{y}{x}=\frac{sin⁡(t)}{cos⁡(t)}$$

Sometimes, it helps to rewrite

$tan⁡(t)$ as $\frac{sin⁡(t)}{cos⁡(t)}$

Simplify:

a. cos(t)∙tan(t)

b. $\frac{(\frac{3}{5})}{3}$ $\frac{tan(x)}{sin(x)}$

Simplify:

c. $\frac{(\frac{1}{3})}{(\frac{1}{6})}$ $\frac{(\frac{1}{cos(x)})}{(\frac{1}{sin(x)})}$

d. Add and simplify

 $\frac{5}{3}+\frac{3}{5}$ $\frac{sin(x)}{cosx}+\frac{cos(x)}{sin(x)}$

e. Subtract and simplify

 $( \frac{x}{y} )^{2}-\frac{1}{y^{2}}$ $tan^{2}(x)-\frac{1}{cos^{2}(x)}$

f. Divide sin2(x) + cos2(x) = 1 by cos2(x) and simplify to discover another Trigonometric Identity.

5. Recall that sine and cosine are co-functions, meaning that the sine of an angle is the cosine of its complement. Let’s see what this means for a particular angle measure x = 30º.

a. Let x equal the angle 30º, and find the following:

1. sin(30º) = \_\_\_\_\_\_
2. complement of 30º is 90º- 30º= \_\_\_\_
3. cos(90º - 30º) = cos( \_\_\_\_ º) =
4. Notice that the sin(30º) = cos(90°-30º)
5. In general, sin(x) = cos(90º-x)

b. Use the sketch of the right triangle to answer the following:

1. Explain why angles A and B are complementary angles. B

 c

 a

1. sin(A)=\_\_\_\_ cos(B)=\_\_\_\_\_ A b
2. sin(B)=\_\_\_\_ cos(A)=\_\_\_\_

c. Now show that sin(x) = cos(90°-x) by graphing:

1. First sketch a graph of y=cos(x - 90°) by using a horizontal shift of the parent function y = cos(x).
2. What function does the graph of y = cos(x - 90°) look like? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. On the same coordinate axis, graph y = sin(x). Do the graphs of y = cos(x - 90°) and y = sin(x) appear to be equal? \_\_\_\_\_\_\_
4. Show that the cos(x - 90°) = cos(90°- x).You may want to use the fact that cosine is an even function and, therefore, cos(x) = cos(-x).

To summarize: We have shown that the graph of y = sin(x) appears to equal the graph of y = cos(x-90º). We have also shown that, cos(x−90º) = cos(90º−x). Therefore, by the transitive property of equality, it seems reasonable to think that cos(90º−x) is equal to sin(x).

A similar demonstration may be used to show that sin(90°-x) = cos(x) (using the fact that y = sin(x) is an odd function and -sin(x-90º) = sin(-1$∙($90°-x))

6. The co-function identities are written:

**sin(90°−*x*) = cos(*x*) cos(90°− *x*) = sin(*x*)**

**sin(**$\frac{π}{2} $**− *x*) = cos(*x*) cos(**$\frac{π}{2}-$***x*) = sin(*x*)**

Fill in the blanks

a. sin(25°) ≈ .4226; cos(65**°) =\_\_\_\_\_\_\_\_**

b. cos(10°) ≈ .9848; sin(\_\_\_\_°) = .9848

c. sin$(\frac{π}{6})$ = cos(\_\_\_\_\_) = \_\_\_\_\_\_\_

d. sin(120°) ≈ −.8660. Use your calculator to find cos(90°- 120°):\_\_\_\_\_\_

e. Do the co-function identities seem to hold for all real numbers, not just acute angles with measures less than π/2?

f. Find cos$(\frac{5π}{3})$ \_\_\_\_\_\_. What is $\frac{π}{2}-\frac{5π}{3}$ ? \_\_\_\_\_\_\_\_\_\_ Find the sine of this number.\_\_\_\_\_

g. Sin($-\frac{3π}{4}$) = cos(\_\_\_\_\_) =\_\_\_\_\_\_

h. Fill in the following blanks with decimal approximations using a calculator in radian mode.

cos(2) = sin(\_\_\_\_\_\_\_\_) = \_\_\_\_\_\_\_\_\_\_\_\_