**Activity 8.2.2 Decomposing Vectors into Components**

Two vectors are equal if they have the same magnitude and the same direction. One convenient way to draw vectors is on a coordinate system from one location to another.

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1. Mark the location of the tails and the heads of vectors $\vec{v} and \vec{w} as ordered pairs$. The head is the tip of the arrow and the tail is position at the opposite end of the vector.
2. How could we determine if $\vec{v}=\vec{w}$? What criteria should we use to establish that two vectors are equal?
3. The magnitude (length) of a vector$\vec{ v}$ is denoted by $\left‖\vec{v}\right‖$. How can we use the coordinate system as a way of deciding if two vectors have the same magnitude?
4. Using the coordinate axes above, find the magnitude $\left‖\vec{v}\right‖$ of $\vec{v}$. Explain how you arrived at your answer.

$\left‖\vec{v}\right‖=$ **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

1. Using the coordinate axes above, find the magnitude $\left‖\vec{w}\right‖$ of $\vec{w}.$ Explain how you arrived at your answer.

$\left‖\vec{w}\right‖=$

1. Compare the magnitudes of vectors $\vec{v} and \vec{w}.$
2. How can the coordinate system be used to determine if two vectors have the same direction?

1. What is the angle between vector $\vec{v}$ and the horizontal axis? Write your answer in degrees and radians.

$θ=$ \_\_\_\_\_\_\_\_\_\_\_\_ degrees

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1. What is the angle between vector $\vec{w}$ and the horizontal axis? Explain your method. Write your answer in degrees and radians.

$θ=$ \_\_\_\_\_\_\_\_\_\_\_\_\_ degrees

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1. Think about the equation $\vec{v}=\vec{w}$and the two vectors above. Are these vectors equal?
2. Does the location of vectors on the coordinate axes influence whether or not the vectors are equal?
3. Let’s use another way to compare two vectors when they are drawn on coordinate axes. Rewrite the ordered pairs for the tail and head of vectors $\vec{v} and \vec{w}.$

$\vec{v}$: Tail = ( , ) Head = ( , )

$\vec{w}$: Tail = ( ) Head = ( , )

1. Subtract the *x*-coordinates and the *y*-coordinates for each vector and write the result as an ordered pair. Are the results the same for each vector?

**Decomposition of Vectors**

The vector $\vec{v}$ from a point $P=\left(x\_{1},y\_{1}\right)$ to a point $Q=\left(x\_{2},y\_{2}\right)$ can be *decomposed* into its horizontal *x*-component, $x\_{2}-x\_{1}$, and vertical *y*-component, $y\_{2}-y\_{1}$. Two vectors $\vec{v} and \vec{w}$ are equal if they have equal *x*-components and *y­*-components.

1. Check whether or not the two vectors $\vec{v} and \vec{w}$ are equal by decomposing them into *x-* and *y-* components.

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1. Check whether or not the two vectors $\vec{v} and \vec{w}$ are equal by decomposing them into *x-* and *y-* components.

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**Representing Vectors as Ordered Pairs and Arrays**

Since all vectors with the same magnitude and direction are equal, we often represent a vector $\vec{v}$ by the ordered pair $\left(v\_{1},v\_{2}\right)$ that would be the location of the head of the vector if the vector’s tail were moved to the origin of the coordinate axes. We can write this ordered pair vertically in brackets as $\left[\begin{matrix}v\_{1}\\v\_{2}\end{matrix}\right]$. This form of representing vectors will help us when we work with matrices.

1. Visually move the tail of vector $\vec{w}$ below to the origin, write its components as an ordered pair in parentheses, and write its components vertically in brackets.

Note: If you have difficulty visually moving the vector, you could instead calculate the components as you did in Questions 14 and 15.

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