**Unit 8: Investigation 2 (3 - 4 Days)**

**Operations with Vectors**

**Common Core State Standards**

N-VM 1Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, ***v***).*,*

N-VM 2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N-VM 3 Solve problems involving velocity and other quantities that can be represented by vectors.

N-VM 4 Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM 5 Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy).

b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).

**Overview**

This investigation begins with the context of a squirrel’s encounter with a car to introduce vectors and operations with vectors. Velocity vectors continue to serve as a model for vector operations throughout the earlier activities. Once students have a firm understanding of vector quantities, they explore how many other common physical quantities have vector attributes.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Given contextual mathematical data, students will be able to recognize when data are vector quantities and suitable for representing with vectors.
* Students will be able to represent vector quantities as directed line segments and use appropriate symbols related to vector quantities ($\vec{v},\left‖\vec{v}\right‖$).
* Students will be able to find the components of a vector by subtracting the coordinates of the initial point from the coordinates of the terminal point and to solve problems involving vector quantities such as velocity using vector components.
* Students will be able to add vectors end to end when represented as directed line segments, add vectors by using components, and add vectors using the parallelogram rule.
* Student will be able to find the magnitude and direction of the sum of two vectors.
* Students will know that $-\vec{u}$ is the additive inverse of $\vec{u}$ and that subtraction of two vectors $\vec{v}-\vec{u}$ amounts to adding the inverse vector:$ \vec{v}+\left(-\vec{u}\right) $and will represent vector subtraction graphically.
* Students will be able to multiply a vector by a scalar and compute the magnitude of a scalar multiple of a vector. ($\left‖c\vec{v}\right‖=\left|c\right|\vec{v}$).

**Assessment Strategies: How will they show what they know?**

* **Exit Slip 8.2.1** assesses student’s ability to decompose vectors into orthogonal components and uses the idea of a resultant vector.
* **Exit Slip 8.2.2** assesses student’s ability to subtract vectors using different strategies.
* **Journal Prompt 1** asks students to write about traveling in a medium when the medium is moving in relation to their own motion.
* **Journal Prompt 2** asks students to add to their story in Journal Prompt1 above with facts about components of the vectors involved and how that affected the story elements.
* **Activity 8.2.1 Vectors and Vector Notation** (Color copies required) introduces students to the context of vector quantities using the context of an animal running to escape being hit by a moving car.
* **Activity 8.2.2** **Vector Components and Decomposition** has students decompose a vector into components and use vector components to understand planar displacement and planar motion.
* **Activity 8.2.3** **Strategies for Adding Vectors** hasstudents practice adding vectors mechanically using three methods. However, they need to understand the meaning of addition as it relates to vector quantities. There are numerous online video resources to help students visualize vector addition.
* **Activity 8.2.3b Practice with Resultant Vectors** provides additional algebraic and graphical problems combining vectors.
* **Activity 8.2.4** Th**e Zero Vector and Subtracting Vectors** has students explore what a zero vector might look like and what its properties would be and how we would write it. Students will subtract vectors by drawing in the plane and then do vector subtraction with the component method.
* **Activity 8.2.5 Scalar Operations on Vectors** has students learn about multiplying a vector by a scalar when a vector is represented by a directed line segment and by an ordered pair.

**Launch Notes**

Begin this investigation with a discussion about movement and speed. Many animals are struck by cars despite being capable of running fast enough to escape. Ask the students to draw a diagram of a moving car and a squirrel running to escape being hit. In addition to knowing how fast the car is moving and how fast the squirrel can run, ask the students what else they need to know to decide the fate of the squirrel?

**Teaching Strategies**

**Activity 8.2.1 Vectors and Vector Notation** introduces students to vector quantities in the context of an animal running to escape being hit by a moving car. You may start by asking students to decide how to represent the car and the running squirrel. Small groups would be ideal for this activity. Some groups may decide to use arrows to represent direction of motion for the car and the squirrel. You might encourage them to explore how the arrow might be able to have more information than the direction of motion. Next, discuss the terms speed and velocity as they relate to this example. Introduce vector notation including notation for the magnitude of a vector. Ask the class how they would precisely measure direction of a vector and how they might represent the magnitude and direction with an arrow (directed line segment). When using physical models for vectors stress that a vector must have a fixed direction to be a vector.

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| **Group Activity 8.2.1** Small groups are ideal for this activity to promote discussion of the different ways this situation can be represented.  |

There should be a preliminary discussion of angle measure with diagrams. Use of technology is encouraged to facilitate conversion between degrees and radians. Students had a lot of practice using dimensional analysis in unit 6 and now our focus is on applications. It is also useful to include compass directions because problems such as flying or sailing are often posed using compass headings. Students are also given a story involving velocity and asked to convert the story into precisely drawn vectors. They should demonstrate an understanding of vector notation, including magnitude. Both degrees and radians should be incorporated for measuring direction.Thehyperlink for this story is: <http://www.hvatoday.org/assets/PDFs/ConnPaddleGuide.pdf>

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| **Journal Prompt 1**Write a paragraph about the experience of traveling in a medium. Specifically think of a situation where you are traveling in one direction, but the medium in which you are traveling is moving in a different direction. Try to think of a different example than the one in the activity 8.2.1. If nothing comes to mind, use the internet to spark ideas, but make up your own story. Students may suggest the wind with a bird or insect or some kind of aircraft traveling in the wind. Students may have observed birds flying in a stiff wind and seeming to move sideways as they are blown off course. They use their inner compass to redirect their flight so as to move towards their destination. Some other examples would be watercraft navigating a current, or scuba divers swimming in a direction to compensate for the ocean current. Another more esoteric example is redirecting a spacecraft to compensate for the gravitational field of some nearby object so that it reaches its intended destination. |

**Activity 8.2.2** **Vector Components and Decomposition** has students learn to decompose a vector into components and use vector components to understand planar displacement and planar motion. This is best accomplished working in groups with large sheets of graph paper or a room with floor tiles. The floor tiles or large graph paper provide a natural orthogonal scale for visualizing a vector as a directed line segment from one point to another. They should discover that the directed line segment extending from the point (0,0) to the point (2,4) has the same magnitude and direction as a directed line segment extending from the point (2,3) to the point (4,7). Introducing the column vector or matrix representation of a vector avoids confusing an ordered pair as a location in the coordinate plane with a vector decomposed into orthogonal components. The vector above would be represented as $\vec{v}=\left[\begin{matrix}2\\4\end{matrix}\right]$ since each directed line segment extends two units in the *x* direction and four units in the *y* direction. It should be stressed that since two equal vectors have the same magnitude and direction their orthogonal components will be equal. When a vector is represented as $\vec{v}=\left(2, 4\right)$ it is understood that (2, 4) is not intended to denote a point in the plane, but represents the components of the vector.

**Differentiated Instruction (For Learners Needing More Help)** All the activities in this investigation lend themselves to group and pair work and you are encouraged to have students work in groups so you can circulate and provide the individual support these learners need.Physical models of vectors and their components and large graph paper can assist with their understanding of operations on vectors.



**Differentiated Instruction (For Advanced Students)** Point out that representing a vector by its components affords a method to determine when two vectors are equal regardless of the dimension of the space of which the vector is an element. Ask students to explain how a directed line segment may not be applicable to a higher dimension space. Have them do an internet search about the topic.

A visual representation is important. Students have seen an analogous situation with the slope of a line. Stress that two vectors may be equal even though they do not have the same starting and ending point as long as they have matching components. After working with displacement, bring in examples of other vector quantities, especially velocity. Remind them of the car and the squirrel. **Exit Slip 8.2.1** can be used at this point to be sure students are comfortable with ordered pair and vertical bracket notation and it returns to the Housatonic River context introduced in Activity 8.2.1.

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| **Journal Prompt 2**Use your story from Journal Prompt 1 about the experience of traveling in a medium. Add a second paragraph that explores the components of the motion of the medium and your motion. Hint example: Running on a track with a strong wind. What happens at various locations along the track?  |

**Activity 8.2.3** **Strategies for Adding Vectors** should be used after students have utilized a resource such as the one mentioned below. Before engaging the students with adding vectors mechanically, they need to understand the meaning of addition as it relates to vector quantities. There are numerous online video resources that help students visualize vector addition, for example [www.youtube.com/watch?v=ayQpSWhr3os](file:///D%3A%5CInvestigations%5CInvestigation%202%5Cwww.youtube.com%5Cwatch%3Fv%3DayQpSWhr3os) uses an airplane flying in the wind as a model for vector addition. This video shows the parallelogram strategy for vector addition. After the students watch the video, give them cut out vectors with wooden staves or other material and have work in groups or pairs to write a sentence about what it means to add two vectors. Then have them brainstorm how to mathematically add vectors. Compare and contrast the three methods for adding vectors (components, end to end, and parallelogram method) and find the magnitude and sum of a resultant vector. Finally have them make two model vectors for the car and squirrel situation and see how the squirrel’s direction decides its ability to escape being hit.

**Activity 8.2.3b Practice with Resultant Vectors** provides additional practice finding the magnitude and direction of resultant vectors.

**Differentiated Instruction (For Enrichment)** Use components to show that, for three arbitrary vectors $\vec{u}+\left(\vec{v}+\vec{w}\right)=\left(\vec{u}+\vec{v}\right)+\vec{w}$and for any two vectors, $\vec{v}+\vec{u}=\vec{u}+\vec{v}.$

**Exit Slip 8.2.2** returns to the canoeing scenario in the Housatonic River context introduced in Activity 8.2.1. It is a somewhat more advanced example. The challenge question is not for all students and should be deleted from the exit slip for most students and it can be used for advanced students as a group activity or homework problem.

**Differentiated Instruction (For Enrichment)** has a more difficult and involved example at the end of exit slip 8.2.3 to challenge more advanced students. It can be used as a group problem or homework problem.

**Activity 8.2.4** **The Zero Vector and Subtracting Vectors has students** explore what a zero vector might look like and what its properties would be and how we would write it. Students should be asked, given a vector $\vec{v}$to find a vector $\vec{u}$so that $\vec{v}+\vec{u}=\vec{0}$**.** Relating this to numbers what is a natural name for this vector $\vec{u}$? What is its magnitude and direction? Most students haven’t yet achieved a complete understanding of subtraction of a real number as being equivalent to adding the additive inverse of a number. This is a good opportunity to show the parallel with subtraction of vectors and to start making a list of properties of real numbers that have analogs with vectors. Students can use the large graph paper to perform subtraction of vectors. Since $\vec{v}-\vec{u} $is defined as $\vec{v}+(-\vec{u})$**,** students should be able to replicate this on graph paper. It is helpful to have them do this first on their own and then draw the procedure on the board with two arbitrary vectors. Students should then do vector subtraction with the component method. First they will find the components of $–\vec{u}$**,** given $\vec{u}$**.** Then perform the subtraction by adding the components of $\vec{v}$to the components of $–\vec{u}$.

**Differentiated Instruction (For Enrichment)** Activity 8.2.4 can be expanded to include situations like $\vec{u}-\left(\vec{v}+\vec{w}\right)$**,** starting with determining what is meant by $-\left(\vec{v}+\vec{w}\right)$**.**

**Activity 8.2.5 Scalar Operations on Vectors** hasstudents learn about scalar operations on vectors. Before beginning, review what students already have learned about the magnitude of a vector. Especially important is the direction of $\vec{-v}$ and magnitude$\left‖\vec{-v}\right‖$. The word scalar is often confusing, so explain the rationale for the term. Next, another short video would be very helpful, for example the following: [www.youtube.com/watch?v=NQevv67hOvQ](file:///D%3A%5CInvestigations%5CInvestigation%202%5Cwww.youtube.com%5Cwatch%3Fv%3DNQevv67hOvQ) Following the video, a hands on activity should incorporate all of the vector operations thus far encountered. One option would be contextual problem involving a boat traveling in a river with a current. There is another video that is helpful for students to visualize this type of moving medium (a river) with an object (a boat) traveling in the medium. [www.youtube.com/watch?v=s1-EmkqW5jQ](file:///D%3A%5CInvestigations%5CInvestigation%202%5Cwww.youtube.com%5Cwatch%3Fv%3Ds1-EmkqW5jQ). A group project would take advantage of various levels of understanding at this stage and help struggling students.

**Closure Notes**

The big idea for investigation 8.2 is that vectors are a different entity than scalars in that they have magnitude and direction. The magnitude is fairly easy to comprehend as a metric, a scalar quantity that applies to the vector’s length. The idea of direction may seem easy to explain, but the devil is in the details. There is no universal direction that is agreed upon and so the idea of direction can be difficult to nail down with students. Some will just accept the idea without question, but others will think about it and come back with some very difficult questions. What is the orientation of our milky way in the universe of galaxies? Who decides about direction? How is it measured? We hope that at the end of this section, students will begin to see the benefit of having vector quantities and that they have a special place in mathematics and that what sets them apart is their magnitude (scalar) and direction.

**Vocabulary**

Component

Decomposition of a vector

Direction

Magnitude

Parallelogram Method of Addition

Resultant vector

Scalar

Vector

**Resources and Materials**

**All activities except activity 8.2.3b should be assigned. Activity 8.2.3b provides additional practice**

Activity 8.2.1Vectors and Vector Notation (Color copies required)

Activity 8.2.2 Vector Components and Decomposition

Activity 8.2.3 Strategies for Adding Vectors

Activity 8.2.3b Practice with Resultant Vectors

Activity 8.2.4 The Zero Vector and Subtracting Vectors

Activity 8.2.5Scalar Operations on Vectors

Physical models for vectors like pieces from a tinker toy set

Large graph paper, markers

Graphing Calculators

Student Journals

Projector for videos

<http://www.hvatoday.org/assets/PDFs/ConnPaddleGuide.pdf>

[www.youtube.com/watch?v=s1-EmkqW5jQ](file:///C%3A%5CUsers%5Cany%5CDocuments%5CGeometry%20and%20Algebra%20II%5CInvestigations%5CInvestigation%202%5Cwww.youtube.com%5Cwatch%3Fv%3Ds1-EmkqW5jQ),

[www.youtube.com/watch?v=NQevv67hOvQ](file:///C%3A%5CUsers%5Cany%5CDocuments%5CGeometry%20and%20Algebra%20II%5CInvestigations%5CInvestigation%202%5Cwww.youtube.com%5Cwatch%3Fv%3DNQevv67hOvQ)

[www.youtube.com/watch?v=ayQpSWhr3os](file:///C%3A%5CUsers%5Cany%5CDocuments%5CGeometry%20and%20Algebra%20II%5CInvestigations%5CInvestigation%202%5Cwww.youtube.com%5Cwatch%3Fv%3DayQpSWhr3os)

Computers

Rulers

Numerous other videos are readily available.