**Activity 8.5.6** **Using Matrices as Transformations of Points, Lines and Various *n*-gons**

In Investigation 8.4.4, we used matrices to represent transformations of vectors (points) in two-space. We learned that multiplying a vector by a matrix gives a new vector, so matrix multiplication effectively moves points to new locations. We noted that this can be undone by multiplying by the inverse matrix, if the matrix has an inverse.

1. In three-space, the point $\left(a,b,c\right)$ in Cartesian coordinates can be considered a vector $\left[\begin{matrix}a\\b\\c\end{matrix}\right] $where the vector is drawn from the origin to the point.

Multiply this vector by the matrix $\left[\begin{matrix}2&0&0\\0&2&0\\0&0&2\end{matrix}\right], so find:\left[\begin{matrix}2&0&0\\0&2&0\\0&0&2\end{matrix}\right]\left[\begin{matrix}a\\b\\c\end{matrix}\right] $

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1. Describe in words what happens to all of three-space in terms of what happens to points.
2. What matrix would reverse this, moving the point $\left[\begin{matrix}2a\\2b\\2c\end{matrix}\right] $back to $\left[\begin{matrix}a\\b\\c\end{matrix}\right]$?
3. Think about this matrix: $\left[\begin{matrix}2&0&0\\0&1&0\\0&0&1\end{matrix}\right]$. Can you find the determinant quickly, just with mental math? Find the determinant.
4. Why does this matrix have an inverse?
5. What is the inverse of $\left[\begin{matrix}2&0&0\\0&1&0\\0&0&1\end{matrix}\right]$
6. What effect does the matrix $\left[\begin{matrix}2&0&0\\0&1&0\\0&0&1\end{matrix}\right]$ have on a point that is considered a vector?

1. What effect does matrix $\left[\begin{matrix}\frac{1}{2}&0&0\\0&1&0\\0&0&1\end{matrix}\right]$ have on a point that is considered a vector?

1. Determine what the $\left[\begin{matrix}2&0&0\\0&3&0\\0&0&4\end{matrix}\right]$ matrix does to the point $\left[\begin{matrix}1\\2\\2\end{matrix}\right]$

1. Use technology to find the inverse matrix for $\left[\begin{matrix}2&0&0\\0&3&0\\0&0&4\end{matrix}\right]$and verify that it moves $\left[\begin{matrix}2\\6\\8\end{matrix}\right]$back to $\left[\begin{matrix}1\\2\\2\end{matrix}\right]$.

In Euclidean geometry, a line is determined by two points. So the line from $\left[\begin{matrix}1\\2\\2\end{matrix}\right]$ to$\left[\begin{matrix}2\\1\\2\end{matrix}\right]$ could be transformed ***all at one time*** by multiplying $\left[\begin{matrix}2&0&0\\0&3&0\\0&0&4\end{matrix}\right]\left[\begin{matrix}1&2\\2&1\\2&2\end{matrix}\right]=\left[\begin{matrix}2&4\\6&3\\8&8\end{matrix}\right]$ and more importantly, ***all points on the original line will be transformed and be on the new line***.

1. Using the inverse of $\left[\begin{matrix}2&0&0\\0&3&0\\0&0&4\end{matrix}\right], verify that multiplying the line \left[\begin{matrix}2&4\\6&3\\8&8\end{matrix}\right] $

by the inverse matrix gives the original line $\left[\begin{matrix}1&2\\2&1\\2&2\end{matrix}\right]$.

1. An entire triangle can be transformed by multiplying a matrix by a transformation matrix.

Sketch the triangle containing the vertices $\left[\begin{matrix}-2\\-1\end{matrix}\right],\left[\begin{matrix}0\\2\end{matrix}\right],\left[\begin{matrix}2\\-1\end{matrix}\right]$

1. Apply the transformation $\left[\begin{matrix}2&1\\-1&3\end{matrix}\right]$to the triangle by multiplying as follows:

$\left[\begin{matrix}2&1\\-1&3\end{matrix}\right]\left[\begin{matrix}-2&0&2\\-1&2&-1\end{matrix}\right]$. Sketch the new triangle.

Check that your result is the new triangle: $\left[\begin{matrix}-5&2&3\\-1&6&-5\end{matrix}\right]. $Draw the new triangle and see what happened to each of the vertices of the triangle.

1. Look at your original triangle. What point is exactly in the center of the base *AC*?
2. Where does the matrix send this point? $\left[\begin{matrix}2&1\\-1&3\end{matrix}\right]\left[\begin{matrix}0\\-1\end{matrix}\right]?$
3. Check to see if this point is exactly in the center of the base of the new triangle.

1. Find the inverse of the transformation matrix.

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1. Check that multiplying by this inverse matrix reverses all of the effects of multiplying by the original matrix. Check to see that multiplying by the inverse moves line *AC* back to its original location.