

# Kindergarten » Introduction

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In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

- 1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as  $5 + 2 = 7$  and  $7 - 2 = 5$ . (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.
- 2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

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## Grade K Overview

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### Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

### Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

### Number and Operations in Base Ten

- Work with numbers 11-19 to gain foundations for place value.

## Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in each category

## Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

## Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.

# Grade 1 » Introduction

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In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

- 1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.
- 2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.
- 3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.<sup>1</sup>
- 4) 4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

## Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

## Number and Operations in Base Ten

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

## Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

## Geometry

- Reason with shapes and their attributes.

## Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.

<sup>1</sup> Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

# Grade 2 » Introduction

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In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

- 1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
- 2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
- 3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
- 4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

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## Grade 2 Overview

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### Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

### Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

## Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

## Geometry

- Reason with shapes and their attributes.

## Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.

# Grade 3 » Introduction

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In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

- 1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
- 2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example,  $\frac{1}{2}$  of the paint in a small bucket could be less paint than  $\frac{1}{3}$  of the paint in a larger bucket, but  $\frac{1}{3}$  of a ribbon is longer than  $\frac{1}{5}$  of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
- 3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
- 4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

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## Grade 3 Overview

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### Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

### Number and Operations in Base Ten

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

### Number and Operations—Fractions

- Develop understanding of fractions as numbers.

### Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

### Geometry

- Reason with shapes and their attributes.

### Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.



# Grade 4 » Introduction

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In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

- 1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
- 2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,  $15/9 = 5/3$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
- 3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

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## Grade 4 Overview

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- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

## Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

## Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

## Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

## Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

## Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.

# Grade 5 » Introduction

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In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

- 1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
- 2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
- 3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

## Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

## Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

## Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

## Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

## Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

## Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.

# Grade 6 » Introduction

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In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

- 1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.
- 2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.
- 3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as  $3x = y$ ) to describe relationships between quantities.
- 4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and

also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability.

Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

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## Grade 6 Overview

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### Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

### The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Multiply and divide multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

### Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

### Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

### Statistics and Probability

- Develop understanding of statistical variability.

- Summarize and describe distributions.

## Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.





# Grade 7 » Introduction

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In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

- 1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.
- 2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.
- 3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.
- 4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal

work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

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## Grade 7 Overview

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### Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

### The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

### Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

### Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

### Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

### Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.

# Grade 8 » Introduction

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In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

- 1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ( $y/x = m$  or  $y = mx$ ) as special linear equations ( $y = mx + b$ ), understanding that the constant of proportionality ( $m$ ) is the slope, and the graphs are lines through the origin. They understand that the slope ( $m$ ) of a line is a constant rate of change, so that if the input or  $x$ -coordinate changes by an amount  $A$ , the output or  $y$ -coordinate changes by the amount  $m \cdot A$ . Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and  $y$ -intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

- 2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.
- 3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-

dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

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## Grade 8 Overview

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### The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

### Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

### Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

### Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

### Statistics and Probability

- Investigate patterns of association in bivariate data.

### Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.

- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.



## Traditional Pathway: High School Algebra I

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

**Critical Area 1:** By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

**Critical Area 2:** In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

**Critical Area 3:** This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

**Critical Area 4:** In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

**Critical Area 5:** In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Units	Includes Standard Clusters <sup>+</sup>	Mathematical Practice Standards
<p><b>Unit 1</b> Relationships Between Quantities and Reasoning with Equations</p>	<ul style="list-style-type: none"> <li>Reason quantitatively and use units to solve problems.</li> <li>Interpret the structure of expressions.</li> <li>Create equations that describe numbers or relationships.</li> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>Solve equations and inequalities in one variable.</li> </ul>	
<p><b>Unit 2</b> Linear and Exponential Relationships</p>	<ul style="list-style-type: none"> <li>Extend the properties of exponents to rational exponents.</li> <li>Solve systems of equations.</li> <li>Represent and solve equations and inequalities graphically.</li> <li>Understand the concept of a function and use function notation.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> <li>Interpret expressions for functions in terms of the situation they model.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p>
<p><b>Unit 3</b> Descriptive Statistics</p>	<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> <li>Interpret linear models.</li> </ul>	<p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p>
<p><b>Unit 4</b> Expressions and Equations</p>	<ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> <li>Write expressions in equivalent forms to solve problems.</li> <li>Perform arithmetic operations on polynomials.</li> <li>Create equations that describe numbers or relationships.</li> <li>Solve equations and inequalities in one variable.</li> <li>Solve systems of equations.</li> </ul>	<p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
<p><b>Unit 5</b> Quadratic Functions and Modeling</p>	<ul style="list-style-type: none"> <li>Use properties of rational and irrational numbers.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul>	

<sup>+</sup>In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.



## Traditional Pathway: Geometry

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas, organized into six units are as follows.

**Critical Area 1:** In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

**Critical Area 2:** Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

**Critical Area 3:** Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

**Critical Area 4:** Building on their work with the Pythagorean theorem in 8<sup>th</sup> grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

**Critical Area 5:** In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

**Critical Area 6:** Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<b>Unit 1</b> Congruence, Proof, and Constructions	<ul style="list-style-type: none"> <li>Experiment with transformations in the plane.</li> <li>Understand congruence in terms of rigid motions.</li> <li>Prove geometric theorems.</li> <li>Make geometric constructions.</li> </ul>	
<b>Unit 2</b> Similarity, Proof, and Trigonometry	<ul style="list-style-type: none"> <li>Understand similarity in terms of similarity transformations.</li> <li>Prove theorems involving similarity.</li> <li>Define trigonometric ratios and solve problems involving right triangles.</li> <li>Apply geometric concepts in modeling situations.</li> <li>Apply trigonometry to general triangles.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p>
<b>Unit 3</b> Extending to Three Dimensions	<ul style="list-style-type: none"> <li>Explain volume formulas and use them to solve problems.</li> <li>Visualize the relation between two-dimensional and three-dimensional objects.</li> <li>Apply geometric concepts in modeling situations.</li> </ul>	<p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p>
<b>Unit 4</b> Connecting Algebra and Geometry through Coordinates	<ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> <li>Translate between the geometric description and the equation for a conic section.</li> </ul>	<p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p>
<b>Unit 5</b> Circles With and Without Coordinates	<ul style="list-style-type: none"> <li>Understand and apply theorems about circles.</li> <li>Find arc lengths and areas of sectors of circles.</li> <li>Translate between the geometric description and the equation for a conic section.</li> <li>Use coordinates to prove simple geometric theorems algebraically.</li> <li>Apply geometric concepts in modeling situations.</li> </ul>	<p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
<b>Unit 6</b> Applications of Probability	<ul style="list-style-type: none"> <li>Understand independence and conditional probability and use them to interpret data.</li> <li>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</li> <li>Use probability to evaluate outcomes of decisions.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

## Traditional Pathway: Algebra II

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions.<sup>2</sup> Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas for this course, organized into four units, are as follows:

**Critical Area 1:** This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

**Critical Area 2:** Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

**Critical Area 3:** In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as *"the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions"* is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

**Critical Area 4:** In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

<sup>2</sup>In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<b>Unit 1</b> Polynomial, Rational, and Radical Relationships	<ul style="list-style-type: none"> <li>Perform arithmetic operations with complex numbers.</li> <li>Use complex numbers in polynomial identities and equations.</li> <li>Interpret the structure of expressions.</li> <li>Write expressions in equivalent forms to solve problems.</li> <li>Perform arithmetic operations on polynomials.</li> <li>Understand the relationship between zeros and factors of polynomials.</li> <li>Use polynomial identities to solve problems.</li> <li>Rewrite rational expressions.</li> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>Represent and solve equations and inequalities graphically.</li> <li>Analyze functions using different representations.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p>
<b>Unit 2</b> Trigonometric Functions	<ul style="list-style-type: none"> <li>Extend the domain of trigonometric functions using the unit circle.</li> <li>Model periodic phenomena with trigonometric function.</li> <li>Prove and apply trigonometric identities.</li> </ul>	<p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p>
<b>Unit 3</b> Modeling with Functions	<ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul>	<p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
<b>Unit 4</b> Inferences and Conclusions from Data	<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on single count or measurement variable.</li> <li>Understand and evaluate random processes underlying statistical experiments.</li> <li>Make inferences and justify conclusions from sample surveys, experiments and observational studies.</li> <li>Use probability to evaluate outcomes of decisions.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

## Integrated Pathway: Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

**Critical Area 1:** By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

**Critical Area 2:** In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

**Critical Area 3:** By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

**Critical Area 4:** This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

**Critical Area 5:** In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

**Critical Area 6:** Building on their work with the Pythagorean Theorem in 8<sup>th</sup> grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<b>Unit 1</b> Relationships Between Quantities	<ul style="list-style-type: none"> <li>Reason quantitatively and use units to solve problems.</li> <li>Interpret the structure of expressions.</li> <li>Create equations that describe numbers or relationships.</li> </ul>	
<b>Unit 2</b> Linear and Exponential Relationships	<ul style="list-style-type: none"> <li>Represent and solve equations and inequalities graphically.</li> <li>Understand the concept of a function and use function notation.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> <li>Interpret expressions for functions in terms of the situation they model.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p>
<b>Unit 3†</b> Reasoning with Equations	<ul style="list-style-type: none"> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>Solve equations and inequalities in one variable.</li> <li>Solve systems of equations.</li> </ul>	<p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p>
<b>Unit 4</b> Descriptive Statistics	<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> <li>Interpret linear models.</li> </ul>	<p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
<b>Unit 5</b> Congruence, Proof, and Constructions	<ul style="list-style-type: none"> <li>Experiment with transformations in the plane.</li> <li>Understand congruence in terms of rigid motions.</li> <li>Make geometric constructions.</li> </ul>	
<b>Unit 6</b> Connecting Algebra and Geometry through Coordinates	<ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

†Note that solving equations and systems of equations follows a study of functions in this course. To examine equations before functions, this unit could be merged with Unit 1.

## Integrated Pathway: Mathematics II

The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I as organized into 6 critical areas, or units. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

**Critical Area 1:** Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 3, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows  $x+1 = 0$  to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

**Critical Area 2:** Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

**Critical Area 3:** Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

**Critical Area 4:** Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

**Critical Area 5:** Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

**Critical Area 6:** In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<b>Unit 1</b> Extending the Number System	<ul style="list-style-type: none"> <li>Extend the properties of exponents to rational exponents.</li> <li>Use properties of rational and irrational numbers.</li> <li>Perform arithmetic operations with complex numbers.</li> <li>Perform arithmetic operations on polynomials.</li> </ul>	
<b>Unit 2</b> Quadratic Functions and Modeling	<ul style="list-style-type: none"> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p>
<b>Unit 3<sup>†</sup></b> Expressions and Equations	<ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> <li>Write expressions in equivalent forms to solve problems.</li> <li>Create equations that describe numbers or relationships.</li> <li>Solve equations and inequalities in one variable.</li> <li>Use complex numbers in polynomial identities and equations.</li> <li>Solve systems of equations.</li> </ul>	<p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p>
<b>Unit 4</b> Applications of Probability	<ul style="list-style-type: none"> <li>Understand independence and conditional probability and use them to interpret data.</li> <li>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</li> <li>Use probability to evaluate outcomes of decisions.</li> </ul>	<p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
<b>Unit 5</b> Similarity, Right Triangle Trigonometry, and Proof	<ul style="list-style-type: none"> <li>Understand similarity in terms of similarity transformations.</li> <li>Prove geometric theorems.</li> <li>Prove theorems involving similarity.</li> <li>Use coordinates to prove simple geometric theorems algebraically.</li> <li>Define trigonometric ratios and solve problems involving right triangles.</li> <li>Prove and apply trigonometric identities.</li> </ul>	
<b>Unit 6</b> Circles With and Without Coordinates	<ul style="list-style-type: none"> <li>Understand and apply theorems about circles.</li> <li>Find arc lengths and areas of sectors of circles.</li> <li>Translate between the geometric description and the equation for a conic section.</li> <li>Use coordinates to prove simple geometric theorems algebraically.</li> <li>Explain volume formulas and use them to solve problems.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

<sup>†</sup>Note that solving equations follows a study of functions in this course. To examine equations before functions, this unit could come before Unit 2.



## Integrated Pathway: Mathematics III

It is in Mathematics III that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions.<sup>3</sup> They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

**Critical Area 1:** In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

**Critical Area 2:** This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

**Critical Area 3:** Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

**Critical Area 4:** In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

<sup>3</sup>In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<p><b>Unit 1</b> Inferences and Conclusions from Data</p>	<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on single count or measurement variable.</li> <li>Understand and evaluate random processes underlying statistical experiments.</li> <li>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</li> <li>Use probability to evaluate outcomes of decisions.</li> </ul>	
<p><b>Unit 2</b> Polynomial, Rational, and Radical Relationships.</p>	<ul style="list-style-type: none"> <li>Use complex numbers in polynomial identities and equations.</li> <li>Interpret the structure of expressions.</li> <li>Write expressions in equivalent forms to solve problems.</li> <li>Perform arithmetic operations on polynomials.</li> <li>Understand the relationship between zeros and factors of polynomials.</li> <li>Use polynomial identities to solve problems.</li> <li>Rewrite rational expressions.</li> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>Represent and solve equations and inequalities graphically.</li> <li>Analyze functions using different representations.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p>
<p><b>Unit 3</b> Trigonometry of General Triangles and Trigonometric Functions</p>	<ul style="list-style-type: none"> <li>Apply trigonometry to general triangles.</li> <li>Extend the domain of trigonometric functions using the unit circle.</li> <li>Model periodic phenomena with trigonometric function.</li> </ul>	<p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p>
<p><b>Unit 4</b> Mathematical Modeling</p>	<ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> <li>Visualize relationships between two-dimensional and three-dimensional objects.</li> <li>Apply geometric concepts in modeling situations.</li> </ul>	<p><b>Look for and express regularity in repeated reasoning.</b></p>

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.