Claim 3: Communicating Reasoning

Gail Pagano
Connecticut State Department of Education

CSDE Assessment Literacy Workshops
November 13, 14, 18, 19, 2013

Smarter Balanced
Assessment Consortium

Contact Information

Gail Pagano:

Gail.Pagano@ct.gov

860-713-6821

Abe Krisst:

Abe.Krisst@ct.gov

860-713-6894

Claim 3: Communicating Reasoning

"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."

Smarter Balanced Depth of Knowledge (DOK) Levels

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	- Recall conversions, terms, facts			
Understand	-Evaluate an expression -Locate points on a grid or number on number line -Solve a one-step problem -Represent math relationships in words, pictures, or symbols	- Specify, explain relationships -Make basic inferences or logical predictions from data/observations -Use models /diagrams to explain concepts -Make and explain estimates	-Use concepts to solve non-routine problems -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible -Explain phenomena in terms of concepts	-Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	-Follow simple procedures -Calculate, measure, apply a rule (e.g., rounding) -Apply algorithm or formula -Solve linear equations -Make conversions	-Select a procedure and perform it -Solve routine problem applying multiple concepts or decision points -Retrieve information to solve a problem -Translate between representations	-Design investigation for a specific purpose or research question - Use reasoning, planning, and supporting evidence -Translate between problem & symbolic notation when not a direct translation	-Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	-Retrieve information from a table or graph to answer a question -Identify a pattern/trend	-Categorize data, figures -Organize, order data -Select appropriate graph and organize & display data -Interpret data from a simple graph -Extend a pattern	-Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence -Generalize a pattern -Interpret data from complex graph	-Analyze multiple sources of evidence or data sets
Evaluate			-Cite evidence and develop a logical argument -Compare/contrast solution methods -Verify reasonableness	-Apply understanding in a novel way, provide argument or justification for the new application
Create	- Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop an alternative solution -Synthesize information within one data set	-Synthesize information across multiple sources or data sets -Design a model to inform and solve a practical or abstract situation

Smarter Balanced Assessment Consortium

Rationale for Claim 3

- Claim 3 refers to a recurring theme in the CCSSM content and practice standards—the ability to construct and present a clear, logical, convincing argument.
  - For older students, this may take the form of a rigorous, deductive proof based on clearly stated axioms.
  - For younger students, this will involve more informal justifications .
- Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide an example, a justification, an explanation, or a counterexample.

Smarter Balanced Assessment Consortium

Targets for Claim 3

Target A: Test propositions or conjectures with specific examples. (DOK 2).

Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (DOK 3, 4).

Target C: State logical assumptions being used. (DOK 2, 3)

Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)

Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)

Target G: At later grades, determine conditions under which an argument *does* and *does not* apply. (DOK 3, 4)

Claim 3 Standards by Grade Level

Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	HS
3.OA.B	4.OA.3	5.NBT.2	6.RP.A	7.RP.2	8.EE.1	N-RN.A
3.NF.A	4.NBT.A	5.NBT.6	6.RP.3	7.NS.A	8.EE.5	N-RN.B
3.NF.1	4.NBT.5	5.NBT.7	6.NS.A	7.NS.1	8.EE.6	N-RN.3
3.NF.2	4.NBT.6	5.NF.1	6.NS.1	7.NS.2	8.EE.7a	A-SSE.2
3.NF.3	4.NF.A	5.NF.2	6.NS.C	7.EE.1	8.EE.7b	A-APR.1
3.MD.A	4.NF.1	5.NF.B	6.NS.5	7.EE.2	8.EE.8a	A-REI.A
3.MD.7	4.NF.2	5.NF.3	6.NS.6		8.F.1	A-REI.1
	4.NF.3a	5.NF.4	6.NS.7		8.F.2	A-REI.2
	4.NF.3b	5.NF.7a	6.EE.A		8.F.3	A-REI.10
	4.NF.3c	5.NF.7b	6.EE.3		8.G.1	A-REI.11
	4.NF.4a	5.MD.C	6.EE.4		8.G.2	F-IF.1
	4.NF.4b	5.MD.5a	6.EE.B		8.G.4	F-IF.5
	4.NF.C	5.MD.5b	6.EE.6		8.G.5	F-IF.9
	4.NF.7	5.G.B*	6.EE.9		8.G.6	G-CO.C
		5.G.4*			8.G.8	G-CO.9
						G-CO.10
						G-CO.11
						A-APR.B
						A-APR.4
						A-APR.6
						A-REI.C
						F-BF.3
						F-BF.4a
						F-TF.1
						F-TF.2
						F-TF.8
						G-CO.A
						G-CO.B
						G.SRT.A
						G.SRT.B

Essential Properties of Claim 3 Items/Tasks

- For the computer-adaptive (CAT) portion of the summative assessment, Claim 3 will be assessed using a combination of :
  - multiple choice, single correct response; multiple choice, multiple correct response; matching tables; equation/numeric; graphing; and fill-in table items/tasks that focus on mathematical reasoning.
- Some items/tasks will require students to construct chains of reasoning without specific guidance being provided throughout the items/tasks.
- Claim 3 items/tasks may involve the application of concepts and procedures across more than one content domain.

Smarter Balanced Assessment Consortium

Essential Properties of Claim 3 Items/Tasks

- Each of the targets should not lead to a separate item/task
- Items and tasks should provide evidence for several of the assessment targets defined for Claim 3.
- Items and tasks will attend to those places in the content standards that call explicitly for communicating mathematical reasoning.
  - Explain
  - Justify
  - Illustrate

Smarter Balanced Assessment Consortium

Mathematical Practices

- The evidence required of students to satisfy Claim 3 centers around specific statements of the ***mathematical practices*** (MP) contained in the CCSSM. Though not exclusive, MP3 and MP6 are particularly relevant for Claim 3 items.

MP3: Construct viable arguments and critique the reasoning of others

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed
  - elementary students construct arguments using objects, drawings, diagrams, and actions..
  - later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions

Smarter Balanced Assessment Consortium

MP6: Attend to Precision

Mathematically proficient students:

- try to communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.
  - In the elementary grades, students give carefully formulated explanations to each other.
  - In high school, students have learned to examine claims and make explicit use of definitions.

Smarter Balanced Assessment Consortium

How are Claim 3 Items Different from Claim 1 Items?

Primary difference lies in the way that students engage with content standards.

For Claim 1 items, students show knowledge and apply concepts and procedures (DOK 1 and 2)

For Claim 3 items, students:

- Analyze and evaluate propositions and conjectures, the results of simulations, and others' solution processes.
- Use their content knowledge to:
  - Support or refute propositions or conjectures with evidence
  - Make conjectures from partial information or simulations
  - Explain flaws in others' thinking or solution processes

Smarter Balanced Assessment Consortium

The Mathematics Assessment

Sample Claim 3 Items

Target A: Test propositions or conjectures with specific examples.

Task Model 1:

Students are given a proposition or conjecture about a real world problem or mathematical phenomenon and are asked to:

- Identify valid tests of a proposition.
- Identify a valid conjecture about the situation or phenomenon.
- Generate a valid test of the proposition.
- Generate a valid conjecture about the situation or phenomenon.

Example Grade 3 Item for Task Model 1 : Test Propositions or Conjectures

William used 6 squares to make the figure shown.

He claims that he can **add exactly 1 more** square to this shape to:

- Increase the perimeter
- Decrease the perimeter
- Keep the perimeter the same as the original perimeter

A. Click to add 1 square to **increase** the perimeter.

B. Click to add 1 square to **decrease** the perimeter.

C. Click to add 1 square to keep the perimeter **the same**.

Example Grade 3 Item for Task Model 1 (Cont.)

William used 6 squares to make the figure shown.

He claims that he can **add exactly 1 more** square to this shape to:

- Increase the perimeter
- Decrease the perimeter
- Keep the perimeter the same as the original perimeter

A. Click to add 1 square to **increase** the perimeter.

B. Click to add 1 square to **decrease** the perimeter.

C. Click to add 1 square to keep the perimeter **the same**.

Exemplar:

A 

B 

C 

Rubric:

(2 points) The student is able to provide an example that supports each conjecture.
(1 point) The student is able to provide two out of three correct examples.
(0 points) The student is unable to provide at least two correct examples.

For A, the perimeter has to be greater than 14 units.
For B, the perimeter of the figure has to be less than 14 units.
For C, the perimeter of the figure has to be equal to 14 units.

Example Grade 8 Item for Task Model 1 : Test Propositions or Conjectures

A student claims:

$\sqrt[3]{b}$ is always less than b

- Enter two values of b that make the student's claim true.
- Enter two values of b that make the student's claim false.

True	False

Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

Task Model 2:

Students are presented with a real world situation or mathematical phenomenon and are asked to:

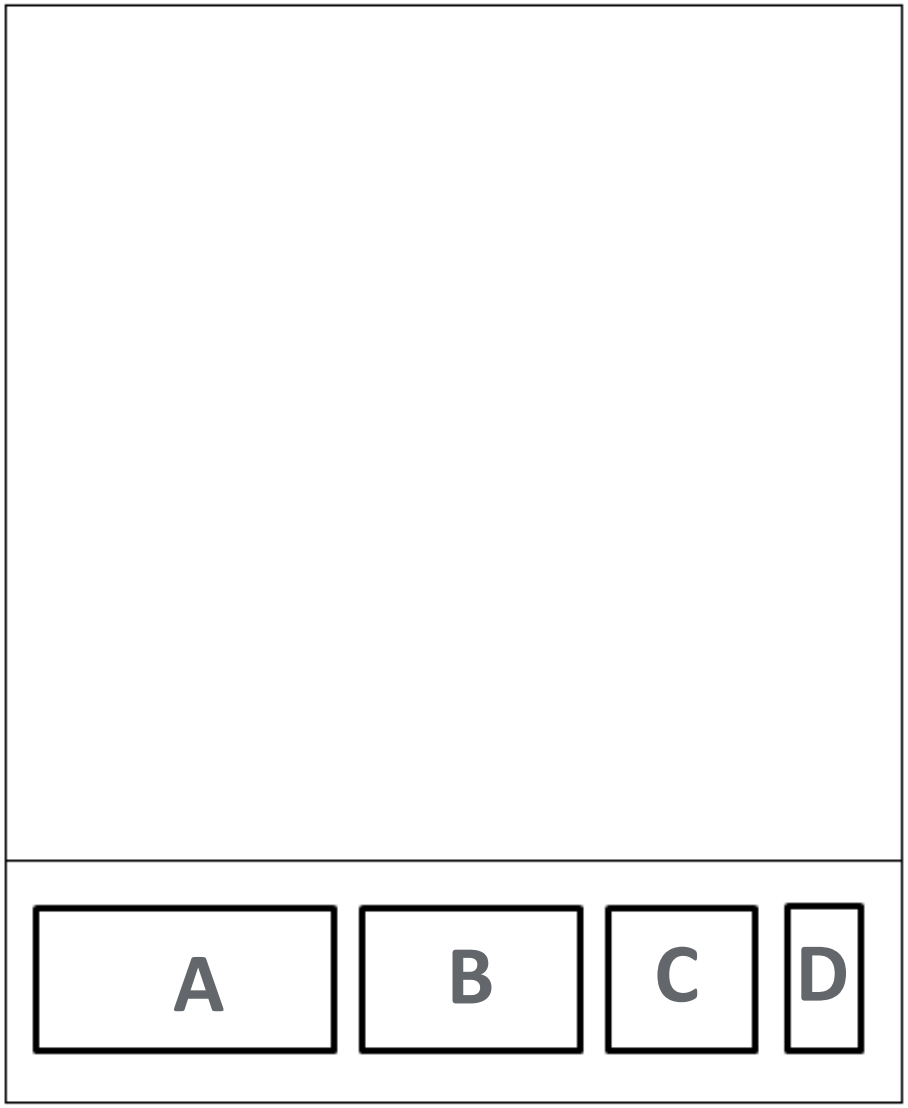
- Generate or identify chains of reasoning that justify a conjecture
- Generate or identify chains of reasoning to disprove a conjecture

Example Grade 5 Item for Task Model 2: Chains of Reasoning

Look at the fraction model shown. The shaded area represents $\frac{3}{2}$.

Robert claims that the only way to model $3 \times \frac{3}{2}$. using rectangles A, B, C, and D, is to use 3 of rectangle B.

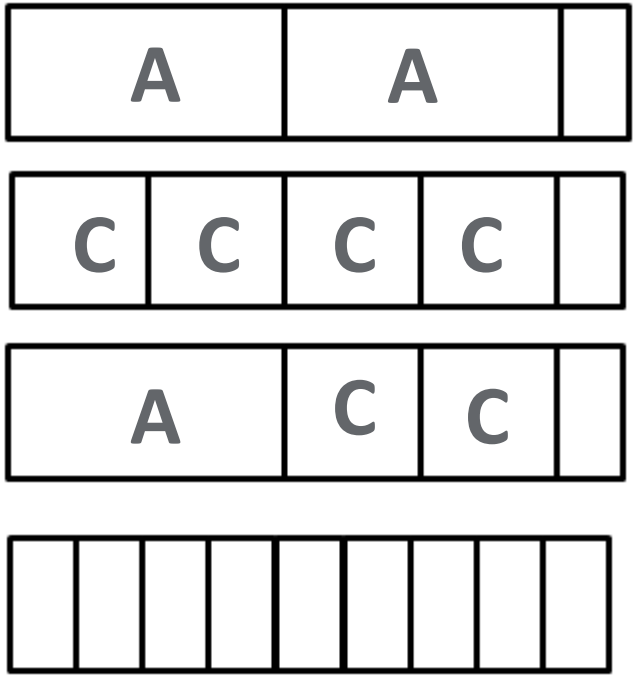
Drag rectangles to the answer space to construct a model to show that Robert's claim is incorrect.



Example Grade 5 Item for Task Model 2: Chains of Reasoning (Cont.)

Rubric (1 point): The student is able to correctly model 9/2 or 4 ½ using a combination of shapes.

Exemplars:



Example HS Item for Task Model 2: Chains of Reasoning

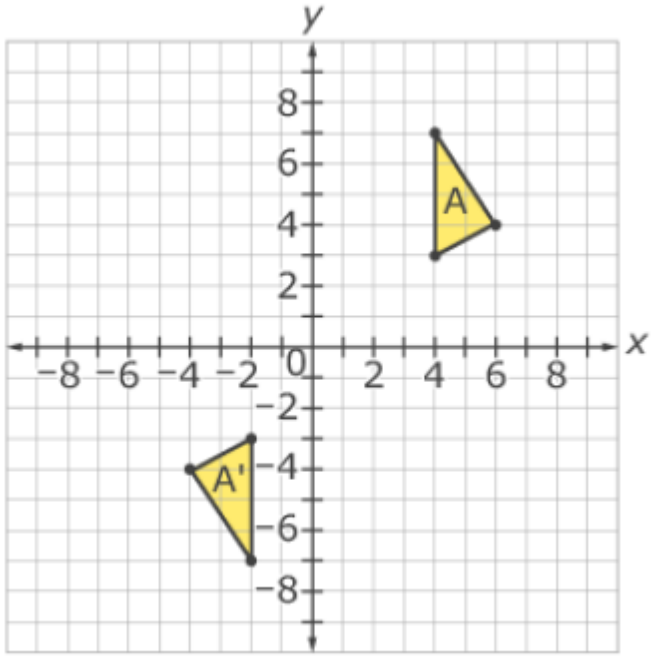
Jose and Tina are writing a program for a computer game. They need to move Figure A to Figure A'.

Jose thinks –

- it will take three separate transformations, and
- there is only one possible way to do it.

Tina thinks –

- there are multiple sets of transformations that will work, and
- it can be done in less than three transformations.



Part A

Describe a set of **three** transformations to support Jose's thinking.

Part B

If possible, provide an example that supports Tina's claim that it can be performed in either **one or two** separate transformations.

(If completed in one transformation, leave the second response box empty.)

Example HS Item for Task Model 2 (Cont.)

Exemplar:

Describe a set of three transformations that support Jose's thinking.

1. reflection across the *x*-axis
2. reflection across the *y*-axis
3. translation 2 units to the right

If possible, provide an example that supports Tina's claim that it can be transformed in either one or two separate transformations.

1. rotation 180 about the origin
2. translation 2 units to the right

OR

1. rotation 180 about the point (1, 0)

Rubric:

(2 points) The student is able to generate three correct translations to support Jose's thinking and one or two transformations to support Tina's thinking. There are multiple correct answers that will need to be reviewed during range finding activities.

(1 point) The student is able to generate correct transformation(s) to support either Jose's thinking or Tina's, but not both.

Target D: Use the technique of breaking an argument into cases

Task Model 3:

Students are given a real world problem or mathematical phenomenon and are asked to:

- Break the phenomenon down into cases.
- Identify cases that support an argument about the phenomenon.
- Generate cases to support an argument about the phenomenon.

Example Grade 4 Item for Task Model 3: Breaking into Cases

- Use this inequality to answer the question.

$$\frac{3}{?} \times 10 < 10$$

Select **all** the numbers that make the inequality true.

A. 2

B. 3

C. 7

D. 9

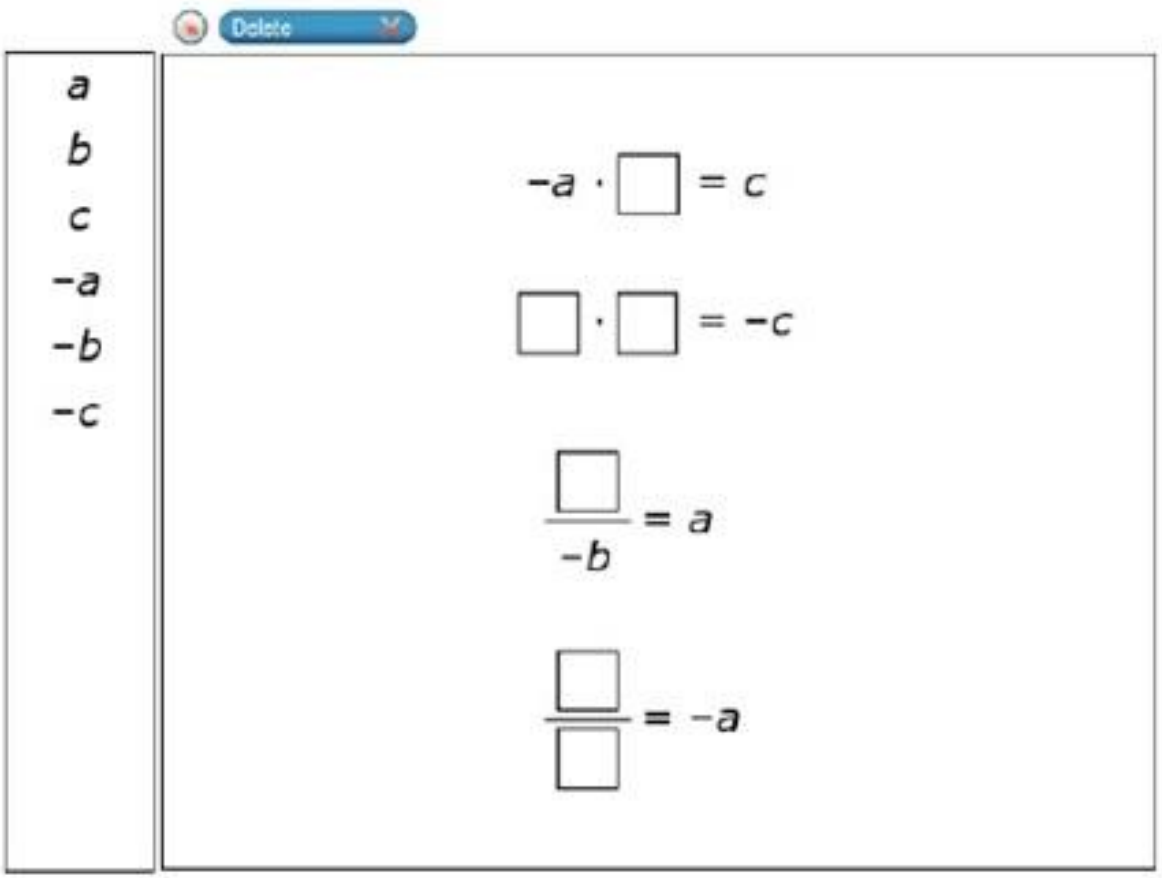
Example Grade 7 Item for Task Model 3: Breaking into Cases

An equation is shown.

$$a \cdot b = c$$

Given this equation, drag one value into each box to complete four different equations.

Assume a, b, and c are not equal to 0.



Example Grade 7 Item for Task Model 3 (Cont.)

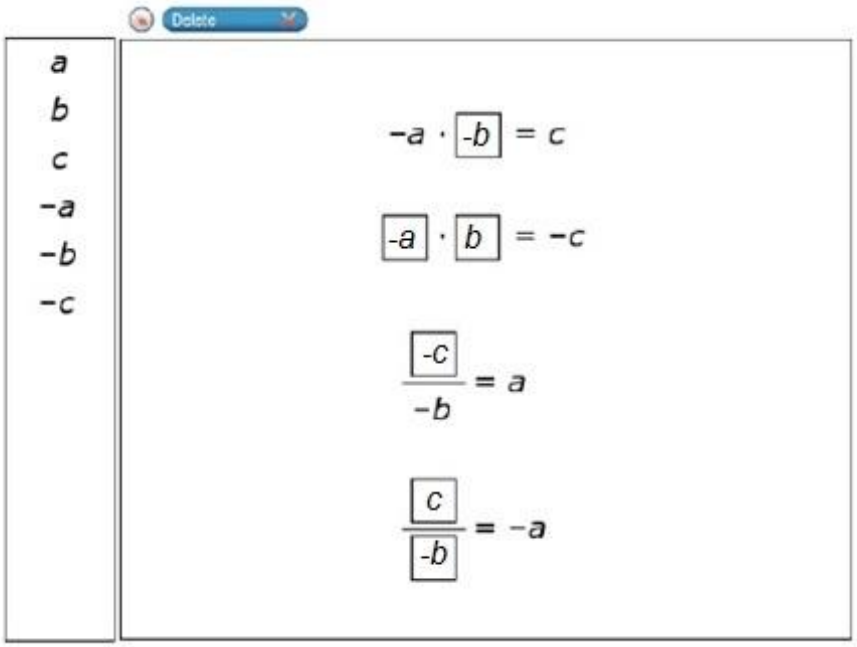
Exemplar:

An equation is shown.

$$a \cdot b = c$$

Given this equation, drag one value into each box to complete four different equations.

Assume a, b, and c are not equal to 0.



Rubric:

(2 points) The student is able to complete all four equations correctly.

(1 point) The student is able to complete 3 out of 4 equations correctly.

Multiple equations are possible

Target G: Determine conditions under which an argument does and does not apply.

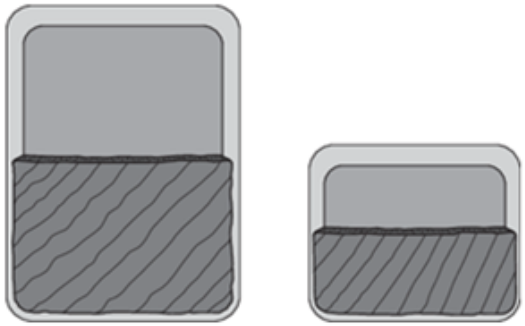
Task Model 4:

Given a real world problem or mathematical phenomenon and a proposition, items may ask students to:

- Explain why a proposition is true or not true.
- Explain the circumstances under which a proposition would be true.

Example Grade 3 Item for Task Model 4: When Does an Argument Apply?

Anton makes brownies for his class using two different baking pans. He has $\frac{1}{2}$ of the brownies left in each pan after school.



$\frac{1}{2}$ $\frac{1}{2}$

- Anton says he has the same amount of brownies in each pan.
- Gail says that Pan X has a greater amount of brownies than Pan Y.

Which student is correct? Click on Anton or Gail.

Anton **Gail**

Click on the statement that explains why the student you selected is correct.

A. Both fractions refer to brownies and $\frac{1}{2} = \frac{1}{2}$ so Anton is correct.

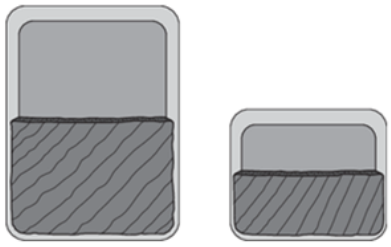
B. The pans would have to be the same size for $\frac{1}{2}$ to be the same in both pans.

C. Since $\frac{1}{2} = \frac{1}{2}$, the amount of brownies in each pan is the same.

Example Grade 4 Item for Task Model 4 (Cont.)

Exemplar:

Anton makes brownies for his class using two different baking pans. He has $\frac{1}{2}$ of the brownies left in each pan after school.



Pan X $\frac{1}{2}$

Pan Y $\frac{1}{2}$

- Anton says he has the same amount of brownies in each pan.
- Gail says that Pan X has a greater amount of brownies than Pan Y.

Which student is correct? Click on Anton or Gail.

Anton | **Gail**

Click on the statement that explains why the student you selected is correct.

A. Both fractions refer to brownies and $\frac{1}{2} = \frac{1}{2}$ so Anton is correct.

B. The pans would have to be the same size for $\frac{1}{2}$ to be the same in both pans.

C. Since $\frac{1}{2} = \frac{1}{2}$, the amount of brownies in each pan is the same.

Rubric:

(1 point) The student can identify which argument is valid in this case

Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is.

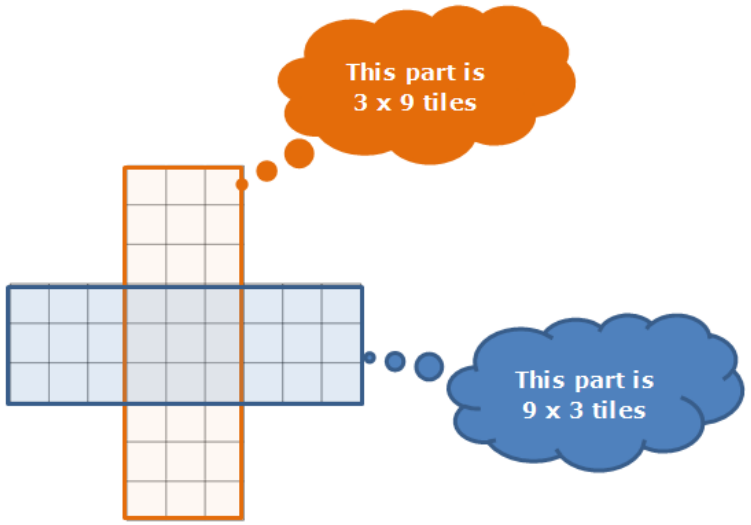
Task Model 5:

Reasoning related to a real world problem or mathematical phenomenon is presented and the task is to:

- Identify flawed or valid reasoning or procedure.
- Explain flawed reasoning.

Sample Grade 3 Item for Task Model 5: Critiquing Others' Reasoning

Tasha is doing an art project with square tiles. This picture shows her design. She thinks…



Tasha says, "I need (3 x 9) + (9 x 3) = 27 + 27 = 54 tiles to make the design."

Which statement explains why Tasha is **not** correct?

A. 27 + 27 does not equal 54

B. (3 x 9) does not equal (9 x 3)

C. Tasha multiplied 9 x 3 incorrectly.

D. Tasha includes the 9 squares in the middle twice*.

Example Grade 7 Item for Task Model 5: Critiquing Others' Reasoning

A shirt is on sale for 10% off $25. Four students attempt to find the new price.

	Valid	Invalid
Tom: $25 \times 0.10 = 2.50$ $25 - 2.50$		
Lucia: 25×0.9		
Ben: $25 - \frac{10}{100}(25)$		
Steve: $25 - .10$		

Click on "valid" if the student's approach is valid. Click on "invalid" if the student's approach is not valid."

Example Grade 8 Item for Task Model 5: Critiquing Others' Reasoning

Kyle had to solve a problem. The problem and his work are shown in the box.

Select the part of Kyle's work that has a mistake.

Select the part of the problem Kyle should read again to fix his mistake.

A company sells baseball gloves and bats. The gloves regularly cost $30 and the bats regularly cost $90. The gloves are on sale for $4 off. The bats are on sale for 10% off. The goal is to sell $1200 worth of bats and gloves each week. Last week, the store sold 14 gloves and 9 bats.

Did the store meet its goal?

1.
$30
− $4
$26

$26
× 14
$364

2.
$90
÷ 0.9
$100

$100
× 9
$900

3.
$900
+ $364
$1264

Rubric: (1 point) The student selects the flawed work

(i.e., $90/0.9 = $100 and highlights "the bats are on sale for 10% off).

Item Quality Criteria

1a. Does the item provide evidence to support the intended claim? Does the item closely align to the claim, target and primary Common Core standard (including cluster level)?

1b. Is this the most appropriate item type to gather evidence to support the target and standard?

1c. Is the item mathematically correct, including its use of precise mathematical language?

1d. Is the item worth asking?

1e. Does the item appear to be accessible to all students? If not, could the item be revised to be made more accessible and still measure the target and standard?

1f. Do the answer choices or rubrics capture the essence of the target and standard?

1g. Is the item/task developmentally appropriate?

Smarter Balanced Assessment Consortium

Accessibility and Accommodations

Accessibility and Accommodations Considerations for Claim 3

- For those students whose disabilities create barriers to development of text for demonstrating reasoning and formation of an argument, it is appropriate to model an argument via symbols, geometric shapes, or calculator or computer graphic programs.

- Access via text to speech and expression via speech to text technology will be important avenues for enabling many students with disabilities to construct viable arguments.

Smarter Balanced
Assessment Consortium

Accessibility and Accommodations Considerations for Claim 3

- ELL students should be provide with different methods and at different levels of linguistic complexity for explaining their ideas.
- Students' engagement in critique and debate should not be limited to oral or written words, but can be demonstrated through diagrams, tables, and structured mathematical responses where students provide examples or counter-examples of additional problems.

Smarter Balanced Assessment Consortium

Questions

Smarter Balanced
Assessment Consortium

Reflect on Guiding Questions

- What do educators need to do to support student learning?
- What do educators need to do to ensure that students are prepared for the Smarter Balanced assessments?

Smarter Balanced
Assessment Consortium